Image enhancement and sharpening

Lecture 6-7
February 19-26, 2008
Procedures of image processing

- Preprocessing
  - Radiometric correction is concerned with improving the accuracy of surface spectral reflectance, emittance, or back-scattered measurements obtained using a remote sensing system. Detector error correction, Atmospheric and topographic corrections
  - Geometric correction is concerned with placing the above measurements or derivative products in their proper locations.

- Information enhancement
  - **Point operations** change the value of each individual pixel independent of all other pixels
  - **Local operations** change the value of individual pixels in the context of the values of neighboring pixels.
  - **Information enhancement** includes image reduction, image magnification, transect extraction, contrast adjustments (linear and non-linear), band ratioing, spatial filtering, Fourier transformations, principle components analysis, image sharpening, and texture transformations

- Information extraction

- Post-classification

- Information output
  - Image or enhanced image itself, thematic map, vector map, spatial database, summary statistics and graphs
1. Image reduction

1. Sampling every other row or column or Nearest Neighbor in ENVI

2. Pixel aggregate (average of 4 pixels)
2. Image magnification

1. Duplicate every row and column or Nearest Neighbor in ENVI

2. Bilinear resampling

3. Cubic resampling
A common interface in ENVI

For Spatial
- Subsetting
- Reduction
- Magnification
3. Transects (spatial profiles)
4. Spectral profiles
5. Contrast Enhancement (stretch)

- Materials or objects reflect or emit similar amounts of radiant flux (so similar pixel value)
- Low-contrast imagery with pixel range less than the designed radiometric range
  - 20-100 for TM less than the designed 0-255
- To improve the contrast:
  - Linear technique
    - Minimum-maximum contrast stretch
    - Percentage linear contrast stretch
    - Standard deviation contrast stretch
    - Piecewise linear contrast stretch
  - Non-linear technique
    - Histogram equalization
- Contrast enhancement is only intended to improve the visual quality of a displayed image by increasing the range (spreading or stretching) of data values to occupy the available image display range (usually 0-255). It does not change the pixel values, unless save it as a new image. It is not good practice to use saved image for classification and change detection.
Minimum-maximum contrast stretch

\[ BV_{out} = \left( \frac{BV_{in} - \min_k}{\max_k - \min_k} \right)^{quant_k} \]

where:
- \( BV_{in} \) is the original input brightness value
- \( quant_k \) is the range of the brightness values that can be displayed on the CRT (e.g., 255),
- \( \min_k \) is the minimum value in the image,
- \( \max_k \) is the maximum value in the image, and
- \( BV_{out} \) is the output brightness value
Percentage linear and standard deviation contrast stretch

- X percentage (say 5%) top or low values of the image will be set to 0 or 255, rest of values will be linearly stretched to 0 to 255
- ENVI has a default of a 2% linear stretch applied to each image band, meaning the bottom and top 2% of image values are excluded by positioning the range bars at the appropriate points. Low 2% and top 2% will be saturated to 0 and 255, respectively. The values between the range bars are then stretched linearly between 0 and 255 resulting in a new image.
- If the percentage coincides with a standard deviation percentage, then it is called a standard deviation contrast stretch. For a normal distribution, 68%, 95.4%, 99.73% values lie in ±1σ, ±2σ, ±3σ. So 16% linear contrast stretch is the ±1σ contrast stretch.
Histogram of Low-contrast Image

Transformation

Histogram of Min-max Linear Contrast-stretched Image

Histogram of Low-contrast Image

Transformation

Histogram of Percentage Linear Contrast-stretched Image
Saturating the water
Stretching the land

Saturating the land
Stretching the water

Special linear contrast stretch
Or Stretch on demand
When the histogram of an image is not Gaussian (bimodal, trimodal, …), it is possible to apply a piecewise linear contrast stretch. But you better to know what each mode in the histogram represents in the real world.
Stretch both land and water
Nonlinear contrast stretch: histogram equalization

- It automatically reduces the contrast in the very light or dark parts of the image associated with the tails of a normally distributed histogram.

- Some pixels that originally have different values are now assigned the same value (perhaps loss information), while other values that were once very close together are now spread out, increasing the contrast between them.
Histogram Equalization Contrast Enhancement

Original Histogram

Frequency, \( f(BV_i) \)

Histogram of Probabilities

Brightness Value, \( BV_i \)

Transformation Function

Equalized Histogram

Brightness Value, \( BV_i \)
Table 8-3. Example of how a hypothetical 64 x 64 image with brightness values from 0 to 7 is histogram equalized.

<table>
<thead>
<tr>
<th>Frequency, $f(BV_i)$</th>
<th>790</th>
<th>1023</th>
<th>850</th>
<th>656</th>
<th>329</th>
<th>245</th>
<th>122</th>
<th>81</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original brightness value, $BV_i$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$L_i = \frac{\text{brightness value}}{n}$</td>
<td>0</td>
<td>0.14</td>
<td>0.28</td>
<td>0.42</td>
<td>0.57</td>
<td>0.71</td>
<td>0.85</td>
<td>1.0</td>
</tr>
<tr>
<td>(n=7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative frequency transformation:</td>
<td>$\sum_{i=0}^{\text{quant}} f(BV_i) / n$</td>
<td>790</td>
<td>1813</td>
<td>2663</td>
<td>3319</td>
<td>3648</td>
<td>3893</td>
<td>4015</td>
</tr>
<tr>
<td></td>
<td>4096</td>
<td>4096</td>
<td>4096</td>
<td>4096</td>
<td>4096</td>
<td>4096</td>
<td>4096</td>
<td>4096</td>
</tr>
<tr>
<td></td>
<td>-0.19</td>
<td>0.44</td>
<td>0.65</td>
<td>0.81</td>
<td>0.89</td>
<td>0.95</td>
<td>0.98</td>
<td>1.0</td>
</tr>
<tr>
<td>Assign original $BV_i$ class to the new class it is closest to in value.</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>
Histogram equalized

Original
6. Band Ratioing

Sometimes differences in brightness values from identical surface materials are caused by topographic slope and aspect, shadows, atmospheric constitutional change, or seasons changes in sun angle and intensity. Band ratio can be applied to reduce the effects of such environmental conditions. In addition, band ratio also help to discriminate between soils and vegetation.

\[
BV_{i,j,ratio} = \frac{BV_{i,j,k}}{BV_{i,j,l}}
\]

where:
- \(BV_{i,j,k}\) is the original input brightness value in band \(k\)
- \(BV_{i,j,l}\) is the original input brightness value in band \(l\)
- \(BV_{i,j,ratio}\) is the ratio output brightness value
spectral ratio model). The best model with single band inputs, which had an \( R^2(\text{adjusted}) = 73.8\% \) and a standard error of \( S = 0.64 \ \mu g/l \) (about 16\% of the total PC range for the July 1, 2000 overpass), is given by the following equation for PC:

\[
PC = 0.78 - 0.0539(B1) + 0.176(B3) - 0.216(B5) + 0.117(B7)
\]

\[\text{(4)}\]

Relative phycocyanin content (PC)

None of the single band relation passed the Durbin-Watson statistical test.

The best spectral ratio model, which had an \( R^2(\text{adjusted}) = 77.6\% \) and \( S = 0.59 \ \mu g/l \) (about 15\% of the total PC range for the July 1, 2000 overpass), is given by

\[
PC \ (\mu g/l) = 47.7 - 9.21(R31) + 29.7(R41) - 118(R43) - 6.81(R53) + 41.9(R73) - 14.7(R74)
\]

\[\text{(5)}\]

Source: Vincent et al., 2004
Applying best equations derived from Landsat 7 to Landsat 5:

Indicating the best spectral ratio model is more robust than the best model derived from single Bands, with regard to changes in sun angle (season), atmospheric transmission, and instrument Settings between Landsat 5 and 7
Fig. 6. Relative phycocyanin content (PC) displayed as red (8.06–9.25 µg/l) to blue (0–5.17 µg/l), from the July 1, 2000 best spectral ratio model, applied to the July 1, 2000, LANDSAT 7 frame. North is toward the top; the whole frame (shown within the black border) covers 185 × 185 km on the ground.

Fig. 7. Phycocyanin content (PC) displayed as red (10.31–15.77 µg/l) to blue (0–2.50 µg/l), from the July 1, 2000 best spectral ratio model, applied to the September 27, 2000, LANDSAT 5 frame. North is toward the top.

Applied to July 1, 2000

Applied to Sep 27, 2000
7. Spatial filtering to enhance low, high frequency detail, and edges

- Spatial frequency is the number of changes in brightness value per unit distance for any particular part of an image. If there are very few changes in brightness value over a given area in an image, this is referred to as a low-frequency area. Conversely, if the brightness values change dramatically over short distances, this is an area of high-frequency detail.

- To see the spatial frequency, we look at the local (neighboring) pixel value changes.

- The spatial frequency may be enhanced or subdued using:
  - Spatial convolution filtering: based primarily on the use of convolution masks
  - Fourier analysis: mathematically separates an image into its spatial frequency components
7.1 Spatial convolution filtering

- A linear spatial filter is a filter for which the brightness value \( BV_{i,j,\text{out}} \) at location \( i,j \) in the output image is a function of some weighted average (linear combination) of brightness values located in a particular spatial pattern around the \( i,j \) location in the input image.

- The process of evaluating the weighted neighboring pixel values is called two-dimensional convolution filtering.
The size of the neighborhood convolution mask or kernel \((n)\) is usually 3 x 3, 5 x 5, 7 x 7, or 9 x 9.

We will constrain our discussion to 3 x 3 convolution masks with nine coefficients, \(c_i\), defined at the following locations:

\[
\begin{array}{ccc}
  c_1 & c_2 & c_3 \\
  c_4 & c_5 & c_6 \\
  c_7 & c_8 & c_9 \\
\end{array}
\]

The coefficients, \(c_i\), in the mask are multiplied by the following individual brightness values \((BVi)\) in the input image:

\[
\begin{array}{ccc}
  c_1 \times BV_1 & c_2 \times BV_2 & c_3 \times BV_3 \\
  c_4 \times BV_4 & c_5 \times BV_5 & c_6 \times BV_6 \\
  c_7 \times BV_7 & c_8 \times BV_8 & c_9 \times BV_9 \\
\end{array}
\]

The primary input pixel under investigation at any one time is \(BV_5 = BV_{i,j}\).
7.1.1 Low frequency (pass) filter: block the high spatial frequency detail, left the low-frequency
- A kernel with small positive values with the same or a little large central value

7.1.2 High frequency (pass) filter: remove the slowly varying components and enhance the high-frequency local variations.
- a kernel with a high central value, typically surrounded by negative weights
This will result in 2 lines and 2 columns smaller than the original image. Software can deal with this problem in different ways (refer to book p277)
7.1.3 Linear edge enhancement

- **Directional**
- **Laplacian**

  - Highlight points, lines, edges, suppress uniform and smooth regions

Directional

<table>
<thead>
<tr>
<th>Vertical edges</th>
<th>Horizontal edge</th>
<th>NW-SE</th>
<th>NE-SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1 0 1</td>
<td>-1 -1 -1</td>
<td>0 1 1</td>
<td>1 1 0</td>
</tr>
<tr>
<td>-1 0 1</td>
<td>0 0 0</td>
<td>-1 0 1</td>
<td>1 0 -1</td>
</tr>
<tr>
<td>-1 0 1</td>
<td>1 0 1</td>
<td>-1 -1 0</td>
<td>0 -1 -1</td>
</tr>
</tbody>
</table>

Laplacian

| 0 -1 0         | -1 -1 -1        | 1 -2 1|
| -1 4 -1        | -1 8 -1         | -2 4 -2|
| 0 -1 0         | -1 -1 -1        | 1 -2 1|
7.1.4 Nonlinear edge enhancement

- Sobel

\[ \text{Sobel}_{5,\text{out}} = \sqrt{X^2 + Y^2} \]
where
\[ X = (BV_3 + 2BV_6 + BV_9) - (BV_1 + 2BV_4 + BV_7) \]
\[ Y = (BV_1 + 2BV_2 + BV_3) - (BV_7 + 2BV_8 + BV_9) \]

\[
\begin{array}{ccc}
-1 & 0 & 1 \\
-2 & 0 & 2 \\
-1 & 0 & 1
\end{array}
\quad \begin{array}{ccc}
1 & 2 & 1 \\
0 & 0 & 0 \\
-1 & -2 & -1
\end{array}
\]

- Robert

\[ \text{Robert}_{5,\text{out}} = X + Y \]
where
\[ X = |BV_5 - BV_9| \]
\[ Y = |BV_6 - BV_8| \]

\[
\begin{array}{ccc}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{array}
\quad \begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0
\end{array}
\]
7.2 Fourier transform

- Spatial domain to frequency domain
- Frequency image contains all information found in the original image
- Frequency image is useful for image restoration (noise remove), filtering (low, high frequency, and edge detection), radiometric correction, image to image registration
- For noise removed in frequency domain, it is much easier to identify and remove than in spatial domain.
When stationary periodic noise is a single-frequency sinusoidal function in the spatial domain, its Fourier transform is bright points. A line connecting the points is always perpendicular to the orientation of the noise lines in the original image.
Application of a Fourier Transform to Landsat Thematic Mapper Data of an Area Near Al Jubail, Saudi Arabia to Remove Striping

Spatial domain to frequency domain

Frequency domain back to spatial domain after removed the noises
Application of a Fourier Transform to Hyperspectral Data of the Savannah River Site

a. Band 5 DAIS 3715 hyperspectral image (centered on 566 nm) of the MWMF at the Savannah River Site. Stripping is noticeable in the enlarged image.

e. Results of applying the cut filter and inverting the Fast Fourier Transform. The before and after enlargement reveals that stripping was substantially reduced.

b. Fourier transform of the band 5 data.

c. Interactive identification of high-frequency noise.

d. Final cut filter used to remove the noise.

After user-defined cut filter, transform back to spatial domain.
A solution to get G is to put the convolution mask at the center of a zero-value image that has the same size of F.
8. Principle Components Analysis (PCA)

- There are large correlations among remote sensing bands. PCA will result in another uncorrelated datasets: principal component images (PCs). PC1 contains the largest variation, PC2 contains the second largest variation, ...
- The first two or three components (PCs) contain over 90% of information from the original many bands. The rest of bands contains the noises. It is a great compress operation, and reducing the dimensionality of the original data.
- The new principal component images that may be more interpretable than the original data.
### Using Covariance matrix

Using Covariance matrix

Is unstandardized PCA

#### Variance Covariance Matrix:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>56.68</td>
<td>34.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>79.43</td>
<td>46.71</td>
<td>68.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>61.49</td>
<td>49.88</td>
<td>69.59</td>
<td>248.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>134.27</td>
<td>85.22</td>
<td>141.04</td>
<td>330.71</td>
<td>568.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>237.2</td>
<td>14.33</td>
<td>22.92</td>
<td>43.62</td>
<td>78.91</td>
<td>42.65</td>
<td>17.78</td>
</tr>
</tbody>
</table>

### Using correlation matrix

Using correlation matrix

Is standardized PCA

#### Correlation Matrix:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>7</th>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>2</td>
<td>0.98</td>
<td>1.00</td>
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<tr>
<td>3</td>
<td>0.95</td>
<td>0.96</td>
<td>1.00</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>4</td>
<td>0.39</td>
<td>0.44</td>
<td>0.53</td>
<td>1.00</td>
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<tr>
<td>5</td>
<td>0.56</td>
<td>0.61</td>
<td>0.71</td>
<td>0.88</td>
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<tr>
<td>6</td>
<td>0.72</td>
<td>0.76</td>
<td>0.84</td>
<td>0.76</td>
<td>0.95</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.56</td>
<td>0.58</td>
<td>0.66</td>
<td>0.66</td>
<td>0.78</td>
<td>0.81</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
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</tr>
</tbody>
</table>

### Univariate Statistics:

<table>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>64.80</td>
<td>25.60</td>
<td>23.70</td>
<td>27.30</td>
<td>32.40</td>
<td>15.00</td>
<td>110.60</td>
<td></td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>10.03</td>
<td>5.54</td>
<td>5.30</td>
<td>17.76</td>
<td>23.85</td>
<td>12.43</td>
<td>4.21</td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>100.93</td>
<td>34.14</td>
<td>68.83</td>
<td>248.40</td>
<td>568.84</td>
<td>154.92</td>
<td>17.78</td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td>51</td>
<td>7</td>
<td>14</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>242</td>
<td>115</td>
<td>131</td>
<td>105</td>
<td>193</td>
<td>128</td>
<td>130</td>
<td></td>
</tr>
</tbody>
</table>
Or using the Correlation matrix replace the Cov (covariance matrix)

\[ EV \cdot Cov \cdot EV^T = \begin{bmatrix}
\lambda_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_{2,2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_{3,3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_{4,4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_{5,5} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{6,6} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \lambda_{7,7}
\end{bmatrix} \]

where \( EV^T \) is the transpose of the eigenvector matrix, \( EV \), and \( E \) is a diagonal covariance matrix whose elements, \( \lambda_{i,i} \), called eigenvalues, are the variances of the \( p \)th principal components, where \( p = 1 \) to \( n \) components. The largest \( \lambda_{i,j} \) will relate to the PC1, then...
<table>
<thead>
<tr>
<th>Component $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>Eigenvectors, $\lambda_p$</td>
</tr>
<tr>
<td>Difference</td>
</tr>
<tr>
<td>Total Variance = 1193.81</td>
</tr>
</tbody>
</table>

Percent of total variance in the data explained by each component:

$$\text{Computed as } \%_p = \frac{\text{eigenvalue } \lambda_p \times 100}{\sum \text{eigenvalue } \lambda_p}$$

For example,

$$\sum \lambda_p = 1010.92 + 131.20 + 37.60 + 6.73 + 3.95 + 2.17 + 1.24 = 1193.81$$

$$= 1$$

Percentage of variance explained by first component = \( \frac{1010.92 \times 100}{1193.81} = 84.68 \)

<table>
<thead>
<tr>
<th>Percentage</th>
<th>84.68</th>
<th>10.99</th>
<th>3.15</th>
<th>0.56</th>
<th>0.33</th>
<th>0.12</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative:</td>
<td>84.68</td>
<td>95.67</td>
<td>98.82</td>
<td>99.38</td>
<td>99.71</td>
<td>99.89</td>
<td>99.99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>Loadings</td>
</tr>
<tr>
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<tr>
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<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>
Correlation, $R_{k,p}$, Between Band $k$ and Each Principal Component $p$

Computed as:  
$$R_{kp} = \frac{a_{kp} \times \sqrt{\lambda_p}}{\sqrt{Var_k}}$$

For example:

$R_{1,1} = \frac{0.205 \times \sqrt{1010.92}}{\sqrt{100.93}} = \frac{0.205 \times 31.795}{10.046} = 0.649$

$R_{5,1} = \frac{0.742 \times \sqrt{1010.92}}{\sqrt{568.84}} = \frac{0.742 \times 31.795}{23.85} = 0.989$

$R_{2,2} = \frac{0.342 \times \sqrt{131.20}}{\sqrt{34.14}} = \frac{0.342 \times 11.45}{5.842} = 0.670$

where:

$a_{k,p}$ = eigenvector for band $k$ and component $p$

$\lambda_p$ = $p$th eigenvalue

$Var_k$ = variance of band $k$ in the covariance matrix

<table>
<thead>
<tr>
<th>Component $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>Band 1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
</tbody>
</table>
Computer pixel values for the new PCs

\[ \text{new}BV_{i,j,p} = \sum_{k=1}^{n} a_{kp}BV_{i,j,k} \]

New pixel value for new PCs

where \( a_{kp} \) = eigenvectors, \( BV_{i,j,k} \) = brightness value in band \( k \) for the pixel at row \( i \), column \( j \), and \( n \) = number of bands.

See an example in the book p299-301
Inverse PC rotation

- Similar as the inverse FFT, to transfer from frequency domain back to the spatial domain, the inverse PC rotation is to transfer the PC images back into their original data space, without including the noise bands, using the same statistics files saved from the forward PC rotation.

- Be sure to use the same covariance matrix or correlation matrix you used for forward PC rotation.
9. Minimum Noise Fraction Transform (MNF)

- It is determine the inherent dimensionality of image data, to segregate noise in the data, and to reduce the computational requirements for subsequent processing
  - The first rotation uses the principal components of the noise covariance matrix to decorrelate and rescale the noise in the data (noise whitening), resulting in transformed data in which the noise has unit variance and no band-to-band correlations
  - The second rotation uses the PCs derived from the original image data after they have been noise-whitened by the first rotation and rescaled by the noise standard deviation. You can divide the images into two parts: large eigenvalues and coherent eigenimages and near-unity eigenvalues and noise-dominated images. Only use the first part for furthering processing.
  - Use estimate noise statistics from data
    - Shift diff subset (optional), selecting a subset that is spectrally uniform, less variation
    - Bands with larger eigenvalues (>1) contain data, others contain noises.
    - Visually examine the bands along with their eigenvalues and find those bands with noises
  - Inverse MNF use only the first part of the MNF bands to remove the noise from the images.
<table>
<thead>
<tr>
<th>Statistic</th>
<th>MNF Noise Statistics File</th>
<th>MNF Statistics File</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>&lt;null&gt;</td>
<td>Of the original input bands</td>
</tr>
<tr>
<td>Maximum</td>
<td>&lt;null&gt;</td>
<td>&lt;null&gt;</td>
</tr>
<tr>
<td>Minimum</td>
<td>&lt;null&gt;</td>
<td>&lt;null&gt;</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>&lt;null&gt;</td>
<td>&lt;null&gt;</td>
</tr>
<tr>
<td>Eigenvalues</td>
<td>Of the noise covariance matrix</td>
<td>Of the second principal components rotation</td>
</tr>
<tr>
<td>Covariance matrix</td>
<td>Of the noise</td>
<td>Composite transformation matrix</td>
</tr>
<tr>
<td>Correlation matrix</td>
<td>Of the noise</td>
<td>Incorrect values. Do not use.</td>
</tr>
<tr>
<td>Eigenvectors</td>
<td>Of the noise covariance matrix</td>
<td>Of the second principal components rotation</td>
</tr>
</tbody>
</table>
10. Image sharpening or data fusion

- Image sharpening: automatically merge a low-resolution color image with a high-resolution grayscale image.

- Data fusion: quantitatively combining or merging data from multiple sources to maximize the extractible information from the data. It is used to merge remote sensing data of different spatial and/or spectral resolutions to create a higher spatial and/or spectral resolution image(s).
  - HSV (hue-saturation-value), IHS (intensity-hue-saturation), HLS (hue-lightness-saturation) color transforms
  - Brovey transform
  - Gram-Schmidt spectral sharpening
  - PC spectral sharpening
  - CN spectral sharpening

- Data fusion can be used to
  - Low spatial resolution with high spatial resolution image to increase spatial resolution
  - Low spatial and high spectral with high spatial and high spectral to increase both spatial and spectral resolution
  - Two complementary but completely different types of remote sensing data (e.g. radar and optical)
Transform an RGB image to HSV (hue, saturation, value) color space. Replace the value band with the high-resolution image, automatically resample the hue and saturation bands to the high resolution pixel size using a nearest neighbor, bilinear, or cubic convolution tech, and finally transform the image back to RGB color space.
Figure 8. False-color radar image of TS06990 (top), sharpened ETM+ image (middle), and fused radar/ETM+ image (bottom) in the left panel. Two subscenes (A and B) in the middle and right panels, respectively. Symbols correspond to features discussed in the text.
Using a mathematical combination of the color image and high resolution data. Each band in the color image is multiplied by a ratio of the high resolution data divided by the sum of the color bands. The function automatically resamples the three color bands to the high resolution pixel size using a nearest neighbor, bilinear, or cubic convolution tech, and finally transform the image back to RGB color space.
Gram-Schmidt spectral sharpening

First, a panchromatic band is simulated from the lower spatial resolution spectral bands. Second, a Gram-Schmidt transformation is performed on the simulated panchromatic band and the spectral bands, where the simulated panchromatic band is employed as the first band. Third, the high spatial resolution panchromatic band is swapped with the first Gram-Schmidt band. Finally, the inverse Gram-Schmidt transform is then applied to form the pan-sharpened spectral bands.
PC spectral sharpening

- A principal components transformation is performed on the multispectral data. The PC band 1 is replaced with the high resolution band, which is scaled to match the PC band 1 so no distortion of the spectral information occurs. Then, an inverse transform is performed. The multispectral data is automatically resampled to the high resolution pixel size using a nearest neighbor, bilinear, or cubic convolution technique.

  - The assumption behind this is that the first PC carries the information common to all bands and is approximately equal to the high space resolution image.