Centripetal Force

Equipment

<table>
<thead>
<tr>
<th>Qty</th>
<th>Item</th>
<th>Parts Number</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Force Sensor, Economy</td>
<td>CI-6746</td>
</tr>
<tr>
<td>1</td>
<td>Photogate</td>
<td>ME-9498A</td>
</tr>
<tr>
<td>1</td>
<td>Large rod</td>
<td>ME-8738</td>
</tr>
<tr>
<td>1</td>
<td>Small rod</td>
<td>ME-8988</td>
</tr>
<tr>
<td>1</td>
<td>Double rob clamp</td>
<td>ME-9873</td>
</tr>
<tr>
<td>1</td>
<td>Rob Base</td>
<td>ME-8735</td>
</tr>
<tr>
<td></td>
<td>String</td>
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</table>

Theory

Let’s consider a mass moving in a circle, with constant radius $r$ with a constant tangential speed $|v|$. This means that at point $P_1$, and at time $t_1$ the mass has velocity $v_1$, but later on at point $P_2$, and time $t_2$ the mass has velocity $v_2$. Now while the direction of the velocity vector clearly changes as the mass moves from the first point to the second, the magnitude of the velocity vector (the speed) stays the same!

$$|v_1| = |v_2| = |v|$$

Even though the magnitudes of the velocity vectors are the same the acceleration they are experiencing can’t be zero because the velocity vector are changing direction.

$$\Delta v = v_2 - v_1 \neq 0$$

It turns out that $\Delta v$ points directly radially inwards to the center of the circle, and that the difference in orientation of the two velocity vectors is identical to $\Delta \theta$ the angular displacement of the mass as it moved from $P_1$ to $P_2$.

If we consider this motion over an extremely small time interval $\Delta t = t_2 - t_1 \to 0 \ s$ then something interesting happens with the geometry of the situation, that being we get to similar triangles.
Where $\Delta s$ is the arc length (linear displacement) traveled by the mass during the time interval $\Delta t$. From the rules of simple geometry we know that the ratios of corresponding sides of similar triangles are equal to each other. Which allows us to write the following equation.

$$\frac{\Delta s}{r} = \frac{\Delta v}{v}$$

Solving this for $\Delta v$ gives us,

$$\Delta v = \frac{\Delta s \cdot v}{r}$$

If we now divide both sides by the time interval we get,

$$\frac{\Delta v}{\Delta t} = \frac{\Delta s \cdot v}{\Delta t \cdot r}$$

Finally, invoking the definitions of acceleration and velocity we obtain

$$a_c = \frac{v^2}{r}$$

$a_c$ is the centripetal acceleration the mass is experiencing as it moves in a circle of radius $r$. Centripetal means ‘center seeking’. The centripetal acceleration always points towards the center of the curvature of the path the mass is traveling on.

From Newton’s Second Law we know that all forces can be written as $F = ma$. That means that the centripetal force acting on the mass causing it move in a circle can be written as;

$$F_c = ma_c = m \frac{v^2}{r}$$
Just like the centripetal acceleration the centripetal force always points to the center of the curvature of the circular path the mass is traveling on.

Any force can act as a centripetal force be it gravity, tension, friction, or a combination there of. The summation of the forces on the mass, acting in the radial direction, collectively are the centripetal force causing the circular motion. One method to determine the centripetal force acting on a mass is to utilize free body diagrams and force summation equations.

Let us consider a mass traveling in a vertical circle attached to a string. Drawing a free body diagram of this mass when it’s at the lowest point of its circular path give us the following.

At this location in the mass’ path the tension $T$ in the string points straight upwards, and the force of gravity $mg$ points straight downwards. From this diagram we write our force summation equation.

$$T - mg = F_c$$

In this simple case we see that the centripetal force is the difference between the tension in the string, and the force of gravity. (Would this be true for any other location?)
Setup

1. Using the given equipment construct the setup as shown in the picture.
   - Make sure that the force sensor’s hook is aimed straight downwards otherwise when collecting data it won’t be measuring the full force vector.
   - Make sure the photogate, and the force sensor are on the same side of the large rod so that when the cylinder mass is hanging from the force sensor it is right between the two vertical arms of the photogate.
   - Also position the photogate such that its infrared beam is aimed at the center of the cylinder.
2. Make sure the PASCO 850 Universal Interface is turned on and connected to the computer.
3. Double click the Capstone software icon to open up the Capstone software.
4. In the Tool Bar, on the left side of the screen, click on the Hardware Setup icon to open up the Hardware Setup window.
5. In the Hardware Setup window there should be an image of the PASCO 850 Universal Interface. If there is skip to step 6.
   - If there isn’t then click on Choose Interface to open the Choose Interface window.
   - In the Choose Interface window choose PASPORT then select Automatically Detect, and then click OK.
6. On the image of the PASCO 850 Universal Interface click on Digital Inputs Ch(1) to open the list of digital sensors, then scroll down and select Photogate (Single Flag).
   - Plug the photogate sensor into Digital Inputs Ch(1). There should now be the photogate icon showing indicating that the photogate is connected to Digital inputs Ch(1).
7. On the image of the PASCO 850 Universal Interface click on Analog Inputs Ch(A) to open the list of analog sensors, then scroll down and select Force Sensor, Economy.
   - Plug the force sensor, economy sensor into Analog Inputs Ch(A). There should now be the Force Sensor, Economy icon showing indicating that the force sensor, economy is connected to Analog Input Ch(A).
   - At the bottom of the screen change the sample rate of the Force Sensor, Economy to 50 Hz.
8. Using a Veneer Caliper measure the diameter of the cylinder mass.
9. In the Tool Bar click on the Data Summary icon to open up the Data Summary window.
In the Data Summary window click on the properties icon that is directly to the right of where it reads One Photogate (Single Flag), to open up the photogate’s properties window.

In the photogate’s properties window change value of the Flag Length (m) to measured diameter of the cylinder mass, then click OK. The diameter must be entered in meters.

In the Data Summary window click on ‘Force (N)’ to make a properties icon appear to its right. Click on that properties icon to open up the force sensor, Economy’s properties window.

In the force sensor, economy’s properties window click on Numerical Format, then change Number of Decimal Places to 3. Then click Ok.

10. Close the Tool Bar.
11. On Page #1 of the QuickStart Templates click on the Two Displays option.
   - Click on the icon in the middle of the top display, select graph, click on the select measurement for the y-axis, and finally select Force (N). The computer will automatically select time (s) for the x-axis.
   - Click on the icon in the middle of the bottom display, select graph, click on the select measurements for the y-axis, and finally select Speed (m/s). The computer will automatically select time (s) for the x-axis.

Procedure

1. Take the cylinder mass off of the force sensor’s hook, and using a mass scale, measure the mass of the cylinder. Record the measurement in the provided table.
2. With the cylinder mass hanging from the force sensor’s hook, and using a ruler, or meter stick, measure the distance from the bottom of the hook to the center of the cylinder mass.
   - Record this distance as R in the provided table. This length will be the radius of curvature of the path of the cylinder mass.
3. With the mass freely hanging from the force sensor’s hook press the Tare button on the side of the force sensor to zero out the reading of the force sensor.
   - This is done to remove the weight of the mass from the measurement the force sensor is making.
   - Since from the theory section we know that $T - mg = F_c$ if we zero out the weight of the mass, $mg = 0$ for the force sensor, then the equation reduces to $T = F_c$.
4. Pull back the cylinder mass about 15 to 20 centimeters, such that when you release it the cylinder will swing back and forth between the two vertical arms of the photogate.
5. Click on the Record button, near the bottom left of the screen, then release the cylinder mass, and let it swing back and forth for about 10 seconds, then click on Stop to stop recording data.
   - The Force vs. Time graph should look like a nice clean sin wave. If it doesn’t you need to delete this data, check the alignment of your experimental setup, and then run the experiment again.
6. For the Force vs. Time graph click on the Highlight Range icon near its top left to make a highlight box appear in the Force vs. Time graph.
   - Rescale the highlight box, and move it show it covers about one complete wave cycle.
7. Near the top left of the Force vs. Time click on the down arrow next to the capital sigma to open up the highlighted data list.
   - Select maximum, and minimum, make sure nothing else is selected, then click on the capital sigma itself to make the selected values appear on the Force vs. Time graph. Record those two values in the provided table.
8. Near the top right of the Speed vs. Time graph click on the Add a Coordinate Tool icon to add a coordinate tool to the Speed vs. Time graph.
   - Use the coordinate tool to find the value of the speed of the mass that corresponds to the same time interval as the wave cycle that you highlighted in the Force vs. Time graph, and record that value in the table provided.
9. Repeat procedure steps 1 – 8 for the second cylinder mass.
Analysis
Tables (20 points)

<table>
<thead>
<tr>
<th></th>
<th>m (kg)</th>
<th>r (m)</th>
<th>Max(m)</th>
<th>Min (m)</th>
<th>v(m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass</td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Aluminum</td>
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</tbody>
</table>

1. Calculate the theoretical value for the centripetal force for the brass mass using the formula $m \frac{v^2}{R}$. (5 points)

2. Find the experimental value of the centripetal force for the brass mass. The amplitude of any simple sin wave can be calculated by dividing the difference between the maximum and minimum by 2. (5 points)

3. Find the % error for the centripetal force of the brass mass. (5 points)
4. Calculate the theoretical value for the centripetal force for the aluminum mass using the formula \( m \frac{v^2}{r} \). (5 points)

5. Find the experimental value of the centripetal force for the aluminum mass. The amplitude of any simple sin wave can be calculated by dividing the difference between the maximum and minimum by 2. (5 points)

6. Find the % error for the centripetal force of the aluminum mass. (5 points)

7. In regards to the force sensor in this experiment what general direction does the centripetal acceleration vector point? (10 points)
8. Using only the equation for centripetal force, show by what factor the force would change if the velocity of the bob was reduced by half, and briefly explain the results. (10 points)

9. For a mass moving in a vertical circle, draw the free body diagram for when the mass is at the highest point of its path, and then write the force summation equation for that free body diagram. (10 points)