

Collisions – Impulse and Momentum

Equipment

Qty	Items	Part Number
1	Collision Cart	ME-9454
1	Dynamics Track	ME-9493
1	Force Sensor	PS-2104
1	Motion Sensor II	CI-6742A
1	Accessory Bracket	CI-6545
1	Mass Balance	SE-8707

Purpose

The purpose of this activity is to examine the relationship between the change of momentum a mass undergoes during an elastic collision and the impulse the mass experiences during that same collision.

Theory

Newton's Third Law tells us that when mass 1 (m_1) exerts a force on mass 2 (m_2) then m_2 must exert a force on m_1 of equal magnitude but opposite in direction. This can be written as a simple algebraic equation;

$$F_{12} = -F_{21}$$

Since Newton's second law tells us that all forces can be written as $F = ma$, where m is the object's mass and a is its current acceleration, we can substitute that in giving us;

$$m_1 a_1 = -m_2 a_2$$

The average acceleration is the change in an objects velocity per unit time, $a_{avg} = \frac{\Delta v}{\Delta t}$, so we can also substitute this in for the two accelerations giving us;

$$m_1 \frac{\Delta v_1}{\Delta t} = -m_2 \frac{\Delta v_2}{\Delta t}$$

You should notice that there are no subscripts on the Δt . The reason there is no subscript on the Δt is because the two masses are exerting forces on each other over the exact same time period. m_1 can't touch m_2 without m_2 touching m_1 , and vice versa. This means we can multiply this equation by Δt to remove it from the equation all together.

$$m_1 \Delta v_1 = -m_2 \Delta v_2$$

Now we can expand the deltas, then distribute the masses and the negative sign, giving us.

$$m_1 v_1 - m_1 v_{1i} = -m_2 v_2 + m_2 v_{2i}$$

Finally, if we now regroup with the initial velocities on left side of the equation and the final velocities on the right side of the equation, we get:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_1 + m_2 v_2$$

This equation is **The Law of Conservation of Momentum** for an elastic collision, and as you have just seen, we can get it directly from Newton's Third Law. The product of a mass and its velocity is called the mass's momentum ($p = mv$), and in the SI system it has the units of kilograms·meters/seconds (kg·m/s). The Law of conservation of Momentum tells us that the sum of the momentums of the two masses before their collision is equal to the sum of their momentums after their collision. This law can be extended for any number of masses interacting with each other.

Going back to Newton's Second Law ($F = ma$), and inserting the definition of acceleration ($a = \frac{\Delta v}{\Delta t}$) we get;

$$F_{avg} = m \frac{\Delta v}{\Delta t}$$

Here F_{avg} is the average force the mass experiences during the time interval Δt . Muliplying the equation by Δt yields;

$$F_{avg} \Delta t = m \Delta v$$

On the right side of the equation if we pull the mass into the delta then we get $\Delta mv = \Delta p$. This means that the product of the average force the mass experiences and the time duration that the mass experiences that force is equal to the mass's change in momentum.

$$F_{avg} \Delta t = \Delta p$$

We will call this product the impulse the mass experiences, and it should be clear that the impulse has the SI units of Newton·seconds (N·s). Now we have the **Impulse-Momentum Theorem**.

$$I = F_{avg} \Delta t$$

In the cases where Δt is a really small interval, then $\Delta t \rightarrow dt$ and the Impulse-Momentum Theorem becomes;

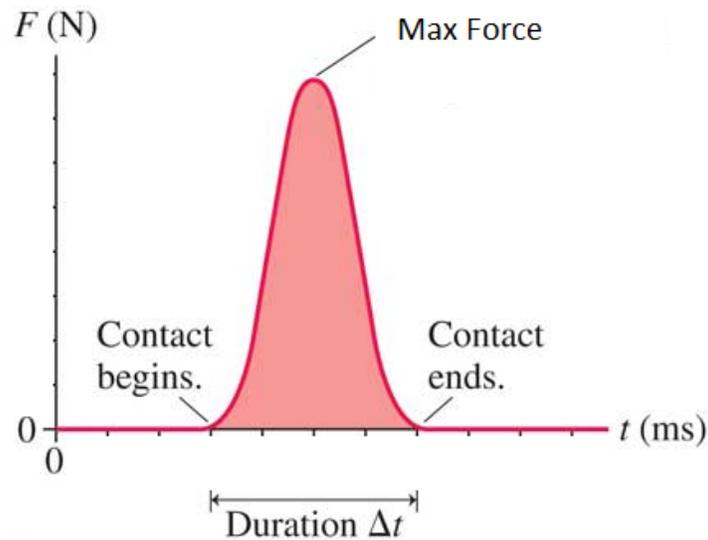
$$I = \int F dt$$

For the above integral the force must be a function of time. For the trivial case where the force is constant, the solution to the integral is

$$I = \int F dt = F \int dt = F \Delta t$$

Such that the impulse is equal to the constant force multiplied by the time interval the force is acting on the mass.

If one were to plot out the force of a collision as a function of time, you get a Force vs. Time graph, which is called an impulse graph. One basic type of an impulse graph has to do with a collision occurring over a very short time period. This type of impulse graph is called Hard Collision, and an example of such a graph is as follows.



Like all impulse graphs, in this graph the 'area under the curve' represents the value of the impulse, but a hard collision impulse graph has two basic defining characteristics that distinguish it from other impulse graphs. First, it is very narrow due to it occurring over a very short time period. Second it has a high peak representing a large maximum force occurring during the collision.

Setup

1. Using the listed equipment, construct the setup as shown with the force sensor attached to the force sensor bracket at one end of the dynamic track, and the motion sensor II attached to the other end. Using a textbook, or something similar, elevate the end of the dynamic track with the motion sensor II attached.



2. Make sure the PASCO 850 Universal Interface is turned on.
3. Double click the Capstone software icon to open up the Capstone software.
4. Plug in the force sensor to the port labelled PASPort 1. The force sensor will automatically be detected by the PASCO 850 Universal Interface.
 - At the bottom of the screen set the Force Sensor sample rate to 500 Hz.
5. In the Tool Bar, on the left side of the screen, click on the Hardware Setup icon to open up the Hardware Setup window.
 - In the Hardware Setup window, you should see an image of the PASCO 850 Universal Interface. Beneath the image of the PASCO 850 Universal Interface, click on the properties icon in the bottom right corner of the window, which will open the properties window.
 - In the properties window you will see Change Sign, select it so that a check sign appears.
 - Click OK to close the properties window.
 - *If the image of the PASCO 850 Universal Interface does not appear, click on the Choose Interface tab in the Hardware Setup window to open the Choose Interface window.*
 - *In the Choose Interface window select PASPORT, then select Automatically Detect, and finally click OK.*
6. On the image of the PASCO 850 Universal Interface click on Ch (1) of the Digital Inputs to open the list of digital sensors.
 - Scroll down and select Motion Sensor II.
 - The motions sensor II icon should now be showing indicating that it is connected to Ch (1), and Ch (2) of the digital inputs.
 - At the bottom of the screen set sample rate of the motion sensor II to 50 Hz.
 - Plug the motion sensor into Ch (1), and Ch (2) of the digital inputs. Yellow in Ch(1), and black in Ch (2).
 - Use the knob on the side of the motion sensor II to make sure it is aimed down the length of the dynamics track.
7. Attach the rubber bumper to the force sensor.
8. In the Tool bar click on the Data Summary icon to open the Data Summary icon.

9. In the Data Summary window, listed under motion sensor II, click on velocity (m/s) to make the properties icon appear directly to the right, then click on the properties icon.
 - In the properties window click on Numerical Format, then set Number of Decimal Places to 3.
10. In the Data Summary window, listed under Force Sensor, click on Force (N) to make the properties icon appear directly to the right, then click on the properties icon.
 - In the properties window click on Numerical Format, then set Number of Decimal Places to 3.
11. Close the Tool Bar.
12. Click on the Two Displays option from the QuickStart templates to open up the two display screen.
 - Click on the display icon for the top display to open the display list, and select Graph. Then for the y-axis click on Select Measurements, and select Force (N).
 - Click on the display icon for the bottom display to open the display list, and select Graph. The for the y-axis click on Select Measurements, and select Velocity (m/s)
 - The computer will automatically select time (s) for the x-axis for both graphs.

Procedure

1. Using a mass scale measure the mass of the dynamics cart and record this mass in the provided chart.
2. Place, and hold the dynamics cart on the dynamics track so that the cart's back is about 20 cm away from the motion sensor II. Make a note of remembering when the dynamic cart is positioned.
3. Press the Tare button on the force sensor to calibrate the sensor.
4. At the bottom left of the screen click on Record to start collecting data, and let go of the dynamics cart, allowing it to be accelerated down the dynamics track till it collides with the force sensor.
 - About one second after the collision click on Stop to stop recording data.
5. Rescale the Force vs time graph (The impulse graph) so that you can clearly see the data points where contact began and contact ended.
6. Click on the Highlight Range icon near the top left of the impulse graph to make a highlight box appear on the impulse graph.
 - Rescale the graph and the highlight box such that the impulse curve is the only thing that is highlighted.
 - Click on the Display area under the curve icon for the impulse graph, and record the measured impulse in the provided data table for I_{area} .
7. Click on the down arrow next to the Σ near the top left of the impulse graph to open up the data list.
 - Select mean, and make sure nothing else is selected
 - Click on the Σ itself to make the data appear on the impulse graph, then record the value for the average force F_{avg} in the table.

8. Click on the Add coordinate tool icon near the top left of the impulse graph to add a coordinate tool to the impulse graph.
 - Using the coordinate tool, identify the time values for when the collision began and when the collision ended, and record those values in the table.
9. Click on the Highlight Range icon near the top left of the velocity vs. time graph to make a highlight box appear on the velocity vs. time graph.
 - Rescale the graph and the highlight box such that the only portion of the velocity vs. time graph that is highlighted corresponds to the same time coordinates of the highlighted portion of the impulse graph.
10. Click on the down arrow next to the Σ near the top of the velocity vs. time graph to open the data window.
 - Select on minimum and maximum, and make sure everything else is not selected.
 - Click on the Σ itself near the top left of the velocity vs. time graph to display the minimum and maximum velocity values on the velocity vs. time graph.
 - Record these values as the initial velocity v_i and the final velocity v_f in the table.
11. Remove the rubber bumper from the force sensor and attach the thin spring.
 - Repeat the entire procedure with the thick spring, then the thin spring making sure to release the cart from the same location as before.

Analysis of Collisions – Impulse & Momentum Lab

Name _____ Group# _____

Course/Section _____

Instructor _____

Table, Mass of Dynamics Cart _____

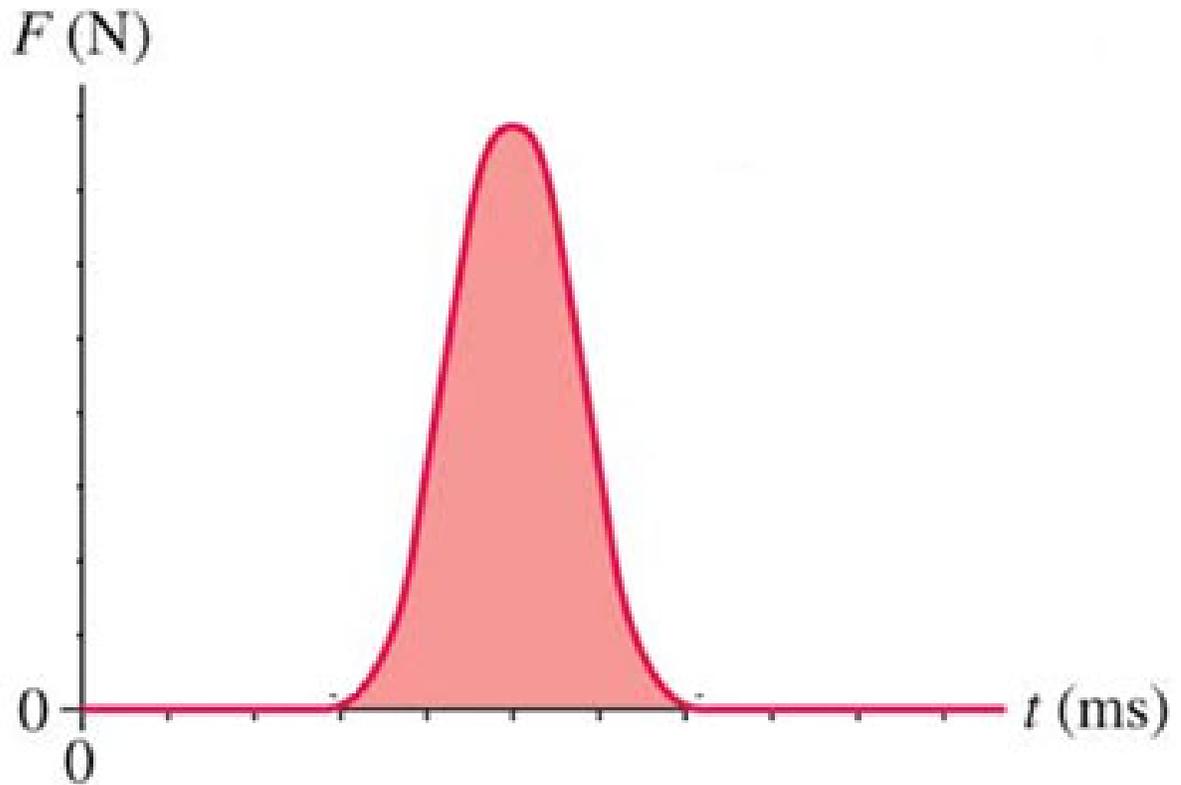
	Rubber Bumper	Thick Spring	Thin Spring
v_i			
mv_i			
v_f			
mv_f			
Δp			
I_{area}			
t_i			
t_f			
Δt			
F_{avg}			

Complete the chart, and show work. (15 points)

1. According to the theory the mass's change in momentum and the impulse experienced by the mass should be the same. What is the % difference between the magnitude of the Δp and I_{area} for each of the separate runs? Does this data seem to support the theory? (10 points)

2. The calculated change in momentum Δp is negative while the impulse given by the area under the curve I_{area} is positive. Why is that? (10 points)

5. Right on top of the given impulse graph of a hard collision sketch the impulse graph of a soft collision with the same magnitude of the impulse. (10 points)



6. Starting from Newton's Third Law $F_{12} = -F_{21}$ derive the Law of Conservation of Momentum for an elastic collision. (10 Points)

