

Standing Waves

Equipment

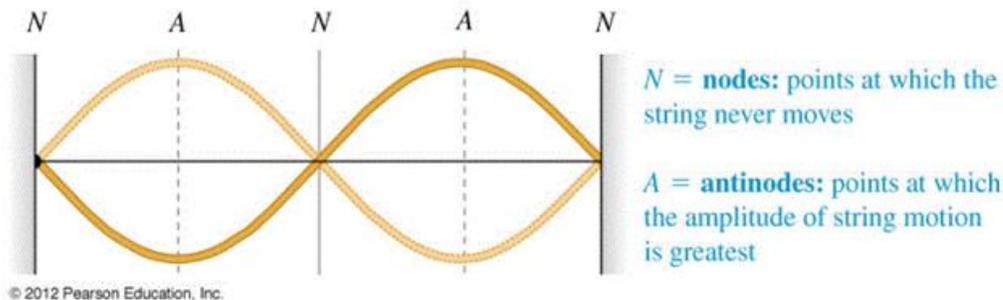
Qty	Items	Parts Number
1	String Vibrator	WA-9857
1	Mass and Hanger Set	ME-8967
1	Pulley	ME-9448B
1	Universal Table Clamp	ME-9376B
1	Small Rod	ME-8988
2	Patch Cords	
1	String	

Purpose

The purpose of this activity is to examine the relationships between the tension in a string, the length density of the string, the length of the string, and the standing waves that can form on the string.

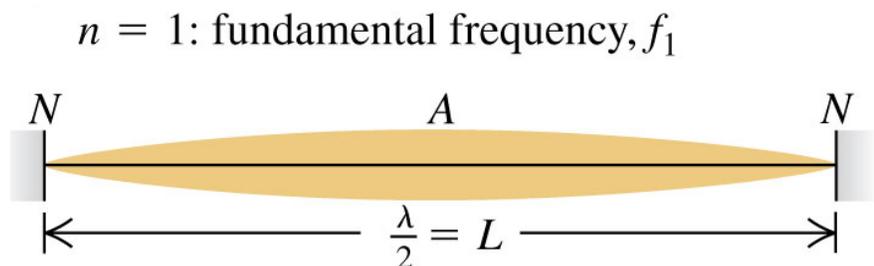
Theory

A standing wave is a wave where the overall pattern does not *appear* to move. For standing wave to form on a string, the basic condition that must be met is that both ends of the string must be fixed in place, never moving themselves. This means that for a string stretched out horizontally, the vertical displacement of each of the two ends of the string must always be zero! In a standing wave, the locations of the string that never move are called nodes, while the locations on the string where the vertical displacement will reach its greatest value are called antinodes.



Using the condition that there must be a node located at each end of the string for a standing wave to form on it, let us construct a relationship between the length of the string and the size of the waves that can form standing waves on that string. Let the length of the string be L , then the largest wavelength that will allow for our

condition to be met is a wave that is twice as long as the string itself, $\lambda = 2L$ which will result in a standing wave with 1 loop to form on the string.



The speed of a wave v is given by the product of its wavelength λ , and its frequency f .

$$v = \lambda f$$

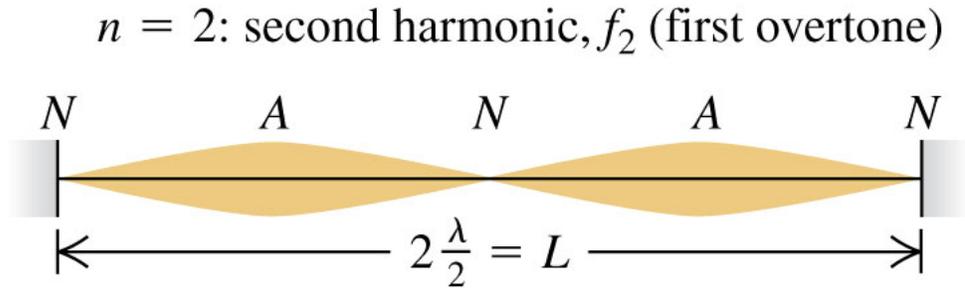
Inserting $2L$ in for the wavelength, and then solving for the frequency gives.

$$f_1 = \frac{v}{2L}$$

This equation gives us the fundamental frequency, or the first frequency that will cause a standing wave to form on the string. This frequency is also called the first harmonic.

Now let us repeat this process to find the second largest wavelength that will form a standing wave on the string of length L . The second largest wavelength that will meet the condition of nodes being at both ends of the string is

one where its wavelength is equal to the length of the string itself $\lambda = L$, which will cause a standing wave with 2 loops to form on the string.



Inserting this into the equation for the speed of a wave, and solving it for the frequency we obtain:

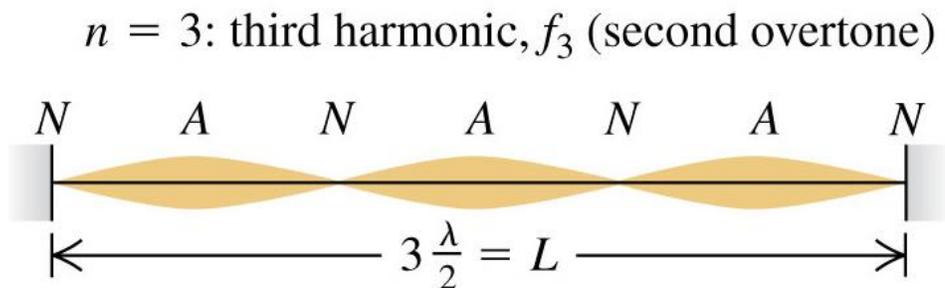
$$f_2 = \frac{v}{L}$$

which is the second frequency that will form a standing wave on this string, aka the second harmonic, aka the first overtone.

Repeating this process one more time to find the third largest wave that will form a standing wave on the string of length L . The wavelength of this wave will be related to the length of the string by $\lambda = \frac{2L}{3}$,

which will cause a standing wave with 3 loops to form on the string.

Inserting this into the equation for the speed of a wave, and solving for the frequency gives,



$$f_3 = \frac{3v}{2L}$$

this, of course, being the third frequency that will form a standing wave on this string, aka the third harmonic, aka the second overtone.

From the equations that give the first three frequencies, one should be able to see a pattern for the equations themselves. The second equation is just 2 times the first equation, and the third equation is just 3 times the first equation.

$$f_2 = \frac{v}{L} = \frac{2v}{2L} = 2 \cdot \left(\frac{v}{2L}\right) = 2 \cdot f_1$$

$$f_3 = \frac{3v}{2L} = 3 \cdot \left(\frac{v}{2L}\right) = 3 \cdot f_1$$

All of the frequencies that will form standing waves on the string follow this basic pattern, such that the n^{th} frequency that will form the n^{th} standing wave pattern on the string is just n times the first frequency f_1 , where n is any counting number.

$$f_n = n \cdot f_1, \quad n = 1, 2, 3 \dots$$

$$f_n = \frac{nv}{2L}, \quad n = 1, 2, 3 \dots$$

Since we know that the speed of a wave on a string is given by $v = \sqrt{\frac{T}{\mu}}$, where T is the tension in the string, and μ is the length density of the string given by $\mu = \frac{m}{L}$. We can insert this into our equation to finally obtain

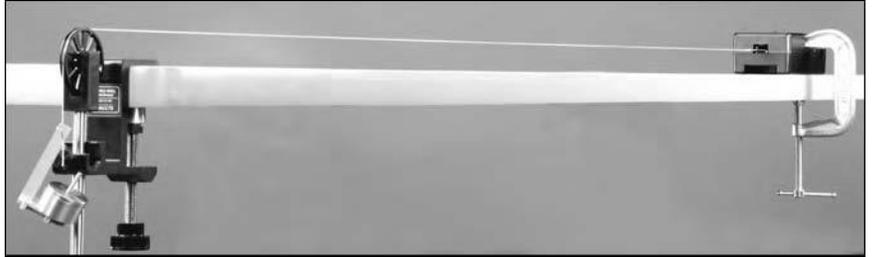
$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}}, \quad n = 1, 2, 3 \dots$$

Setup

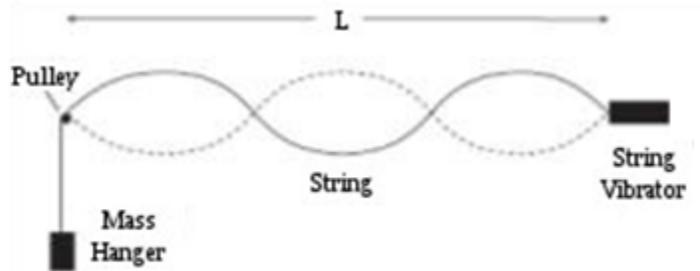
1. Double click the Capstone icon to open up the PASCO Capstone software.
2. In the Tool Bar on the right side of the screen, click on the Hardware Setup icon to open the Hardware Setup window.
 - In the Hardware Setup window there should be an image of the PASCO 850 Universal Interface. If there is, skip to step 3. If there isn't, click on the Choose Interface tab to open the Choose Interface window, then select PASPORT, automatically detect, and finally click OK.
3. On the image of the PASCO 850 Universal Interface click on OUTPUT Ch (1), and then select Output Frequency Sensor.
4. In the Tool Bar, click on the Signal Generator to open the signal generator window.
 - Click on 850 Output 1, which will open up the properties of Output Source Ch (1).
 - Set the wave form to sin.

- Click on the push pin icon near the top right of the signal generator window to rescale the main window so you can keep the signal generator open while you perform the experiment.
5. Use two patch cords to connect Output Source Ch (1) to the string vibrator.
 6. Using the provided equipment construct the setup as seen in the provided picture.

- The length between the string vibrator, and the detachable pulley should be **AT LEAST** one meter.
- The length of the string should be horizontal.
- Don't plug in the string vibrator yet.
- The C-clamp needs to be tight enough that it will hold the string vibrator in place, but not too tight that you crack the plastic case of the string vibrator.
- At this point the exact mass hanging from the hook is unimportant.



7. Using a measuring stick, measure the length L of your string. Please note that L is not the entire length of the string, but the distance between the pulley and the front edge of the metal 'blade' the string is tied to. Record your value for L in the table for string.



8. Take a long length of the same type of string you are using, 3 to 4 meters, and using a mass scale measure the mass m of the string. Then record that value in the table provided.
 - Using a measuring stick measure the length l of this same string. Then record this value in the table for string.
 - Using your values for m and l , calculate the length density $\mu = \frac{m}{l}$ of the type of string you are using, and then record that value in the table for string.

Procedure: Part 1

1. Including the mass of the mass hanger, have 100 grams hanging from the end of the string.
2. In the signal generator window set the frequency of $f = 60.0 \text{ Hz}$, then set the amplitude to 1 V.
3. In the signal generator window, click the on tab to start the string vibrator to start vibrating.
4. Change the frequency till you find the frequency that allows for one standing loop, $n = 1$, to form on your string. Record this frequency in the table provided.
 - You will have to turn the string vibrator off, and then back on to reset the frequency each time.
5. Find the frequencies that yield the standing wave patterns that correspond to $n = 2$, $n = 3$, $n = 4$, $n = 5$, and record them in the table provided.

- In the signal generator window, you will have to increase the amplitude of the wave to better see the standing wave patterns for the higher frequencies.

Procedure: Part 2

1. In the signal generator window, set the frequency of $f = 60.0 \text{ Hz}$.
2. In the signal generator window click the on tab to start the string vibrator to start vibrating.
 - Adjust the mass on the hook by adding or subtracting mass till a standing wave of 2 loops form on the string. (The exact mass will depend on the length L and the length density μ of the string you used.)
 - Once a standing wave of 2 loops has formed on the string, record the total hanging mass as m_2 in the chart provided. Remember the hook itself is 5 grams and needs to be included in the total mass.
3. Calculate the tension, T , the force in the string created by the hanging mass, then record that tension in the table provided.
4. Calculate the speed of the standing wave using the equation $v = \sqrt{\frac{T}{\mu}}$, then record that as the theoretical speed v_T in the tables provided.
5. Calculate the experimental wave speed by using the equation $v = \lambda f$, then record the experimental speed v_E in the tables provided.
 - The wavelength λ is always the total length of 2 loops.
6. Calculate the % error between the theoretical and experimental values of the speed of the standing wave with 2 loops.
7. Repeat Step 4 through 8 for 3, 4, and 5 loops forming on your string, and record the total hanging mass for each case in the provided tables.

Analysis of Standing Waves Lab

Name _____ Group# _____

Course/Section _____

Instructor _____

Table for String (5 points)

Table Part 1 $m = 100\text{ g}$ (5 points)

$m(kg)$	
$l(m)$	
$\mu\left(\frac{kg}{m}\right)$	
L	

n	$f_n(Hz)$
1	
2	
3	
4	
5	

Table Part 2

$f = 60\text{ Hz}$

	m_2	m_3	m_4	m_5
n	2	3	4	5
$m(kg)$				
$T(N)$				
$v_T\left(\frac{m}{s}\right)$				
$v_E\left(\frac{m}{s}\right)$				
%error				

Sample Calculations for table 2: (15 points)

1. For part 1, calculate the theoretical frequency for the standing wave that corresponds to $n = 1$ (5 points)
2. Calculate the % error between your experimental frequency for $n = 1$, and the theoretical frequency. (5 points)
3. According to the theory all the higher frequencies that form standing waves on a string, given identical conditions, should all be whole number multiples. Does your data support this theory? If not, what are some reason why it doesn't? (5 points)
4. From the data from table 2, and using Excel or similar program, graph T (tension) vs n, with the trendline, and show equation on the graph. Describe the shape of the graph. (10 points)
5. Using algebra show that the tension can be written as $T = (4\mu f^2 L^2) \frac{1}{n^2}$ (5 points)

6. Using Excel or a similar program graph T vs $(1/n^2)$, with the trendline, and show the equation on the graph. (5 points)
 - a. From the slope of the graph, calculate the density of the string. (10 points)
 - b. Compare this value to the value you calculated from your measured values using % difference. (10 points)
7. If the length density of the string were doubled, how would the frequency change? (5 points)
8. Write a general statement that describes how the frequency of vibration of a standing wave relates to the number of segments of the standing wave. (5 points)
9. What are a few reasons that account for our percent error? (5 points)
10. Do the results of our experiment confirm the theory? Justify your answer. (5 points)