

Vector Addition

Equipment List

Qty	Item	Part Number
1	Force Table	ME-9447B
1	Mass and Hanger Set	ME-8979
1	Carpenter's level	
1	String	

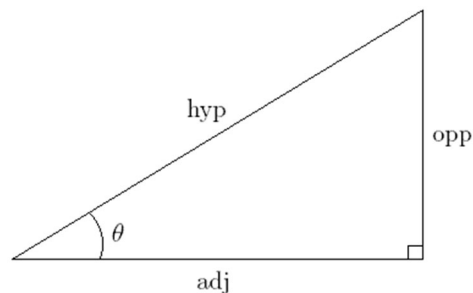
Purpose

The purpose of this lab is for the student to gain a better understand of the basic properties of vectors, and some simple vector mathematics. The student will see how vectors are related to the Right Triangle, and how vectors can be described in both the Rectangular Coordinate system, and the Polar Coordinate system. Also, the student will see how to convert a vector from one coordinate system to the other, and then back again. By adding different combinations of force vectors, and adding those vectors by different means, the student's understanding of vectors, and vector addition, should be improved.

Theory

The Right Triangle, and Simple Trig Functions

A triangle is a three sided, closed, geometric shape, whose three internal angles sum to 180° . A Right Triangle is a triangle that one of its internal Angles is 90° , which is called a right angle. If we pick one of the other two internal angles of a right triangle, and call it ϑ (theta), then we are able to label the three sides of the right triangle in reference to the angle ϑ .



The three sides are:

1. Hypotenuse (hyp) – The side that is opposite the right angle, and it is ALWAYS the longest of the three sides.
2. Adjacent (adj) – This along with the hypotenuse forms the angle ϑ .
3. Opposite (opp) – The side that is opposite of the angle ϑ .

Trig Functions relate the value of the angle ϑ to the value of the ratio of the lengths of two of the sides of a right triangle. There are three basic Trig Functions.

1. $\sin(\vartheta)$ – The sin function relates the value of ϑ to the value of the ratio of the opposite side and the hypotenuse.

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$$

2. $\cos(\vartheta)$ – The cos function relates the value of ϑ to the value of the ratio of the adjacent side and the hypotenuse.

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$$

3. $\tan(\vartheta)$ – The tan function relates the value of ϑ to the value of the ratio of the opposite side and the adjacent side.

$$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

There are also three basic Inverse Trig Functions. As the name suggest the Inverse Trig Functions do the exact opposite of what the Trig Functions do. As the Trig Functions relate the value of the angle ϑ to the value of a ratio of two sides of a right triangle, the Inverse Trig Functions relate the value of the ratio of two sides of a right triangle to the value of the angle ϑ .

1. $\sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right)$ – The inverse sin function relates the value of the ratio of the opposite side of the right triangle, and the hypotenuse.

$$\sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right) = \theta$$

2. $\cos^{-1}\left(\frac{\text{adj}}{\text{hyp}}\right)$ – The inverse cos function relates the value of the ratio of the adjacent side of the right triangle, and the hypotenuse.

$$\cos^{-1}\left(\frac{\text{adj}}{\text{hyp}}\right) = \theta$$

3. $\tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right)$ – The inverse tangent function relates the value of the ratio of the opposite side of the right triangle, and its opposite side.

$$\tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right) = \theta$$

The **Pythagorean Theorem** tell us that the hypotenuse is equal to the square root of the sum of the squares of the two other sides of the right triangle. Usually written as such:

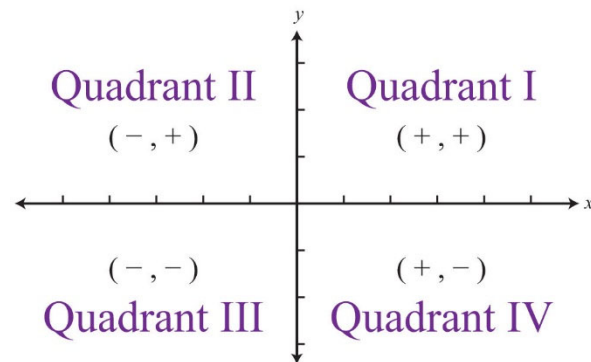
$$r = \sqrt{x^2 + y^2}$$

Coordinate Systems

There are two basic coordinate systems we will be working with in this lab: the rectangular coordinate system, and the polar coordinate system.

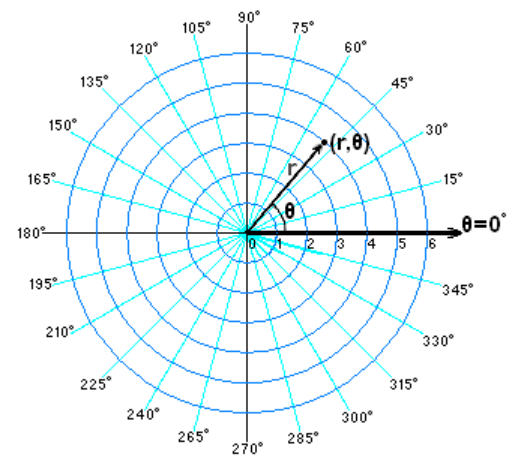
The rectangular coordinate system has the following properties.

1. It has two primary lines, called the Axes that intersect at right angles. The point these two lines intersect is called the Origin.
 - The horizontal axis is called the X-Axis.
 - The vertical axis is called the Y-Axis.
 - The Axes separate the rectangular coordinate system into four regions called quadrants.
2. A point (location) in the coordinate system is identified by a pair of numbers, called an ordered pair.
 - The x-value is listed first, and the y-value is listed second. (x, y)
 - The Origin has the value of $(0, 0)$.
 - The x-values are positive to the right of the Origin, and negative to the left of the Origin.
 - The y-values are positive above the Origin, and negative below the Origin.



Polar coordinate system has the following properties.

1. Centered on a point called the Origin.
2. All points (locations) are identified by two numbers, called an ordered pair.
 - The first number is the point's straight line distance from the Origin, and normally denoted by ' r '.
 - The second number is the point's orientation (direction), normally denoted by ' θ '. Unless otherwise stated, the orientation is taken to be measured from what would be the x-axis in the rectangular coordinate system.
 - Collectively this ordered pair is written as such (r, θ) .
 - A counterclockwise rotation results in a positive value for the orientation. While a clockwise rotation results in a negative value for the orientation.

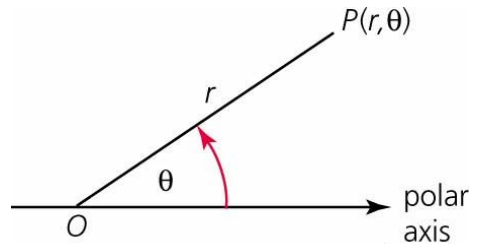


Converting between coordinate systems

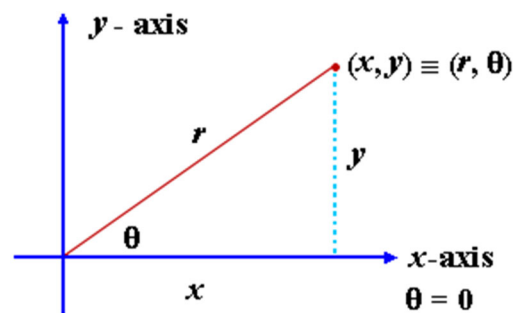
Sometimes we have a point that is described by one coordinate system, but for various mathematical reasons we need it to be described by another coordinate system. So we must convert between coordinate systems. We can, and will, use the properties of the Right Triangle to convert between our polar and rectangular coordinate systems.

Converting from polar coordinate system to a rectangular system.

1. Let us assume we have a point in the polar coordinate system described by the order pair (r, ϑ) , where ϑ is measured from what would be the x-axis in the rectangular coordinate system.



2. If we extend a line straight down from that point to the x-axis it forms a right triangle with the length r playing the role of the hypotenuse in the triangle.



3. In this simple example, the x-leg of our right triangle is the adjacent side, and the y-leg is the opposite side.

4. Applying the trig functions to this we get the following:

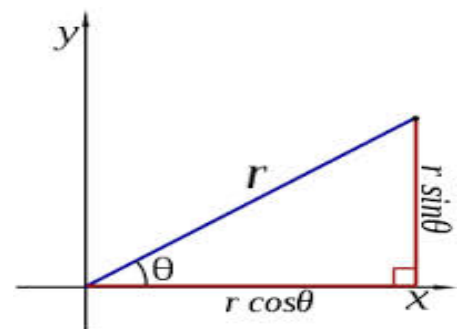
$$\cos(\theta) = \frac{x}{r} \text{ and } \sin(\theta) = \frac{y}{r}$$

If we multiply either by r we get:

$$r \cos(\theta) = x \text{ and } r \sin(\theta) = y$$

- Resulting in the conversion from polar to rectangular coordinates.

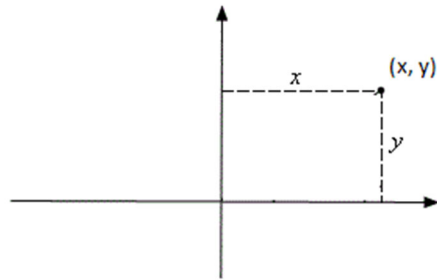
$$(r, \theta) = (r \cos(\theta), r \sin(\theta)) = (x, y)$$



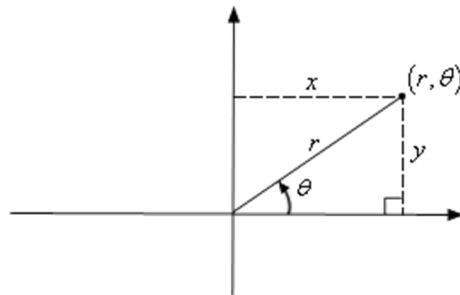
- Please note that the conversion method described assumes that ϑ is measured from the x-axis. If ϑ is measured from the y-axis you get a slightly different conversation relationship. (Try it, and see what you get)

Converting from rectangular coordinates to polar coordinates.

1. Let us assume we have a point in the rectangular coordinate system described by (x, y)



2. Extend a line directly from the origin to the point (x, y) . Call this line r , and it will form an angle θ with the x -axis. Finally, a line that runs vertically from the point (x, y) to the x -axis will complete a right triangle.



3. If we apply the properties of the right triangle to this, we get the following:

$$r = \sqrt{x^2 + y^2} \quad \tan^{-1}\left(\frac{y}{x}\right) = \theta$$

- Resulting in converting to polar coordinates.

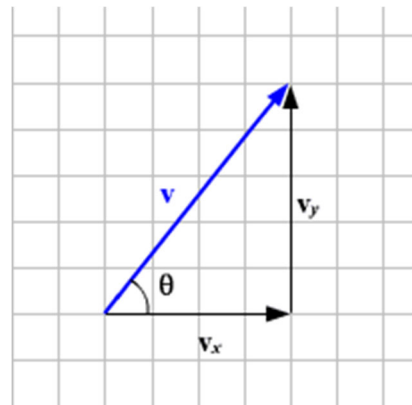
$$(x, y) = \left(\sqrt{x^2 + y^2}, \tan^{-1}\left(\frac{y}{x}\right)\right) = (r, \theta)$$

- Please note getting the orientation θ can be a bit tricky due to the possible signs of the rectangular coordinates, and the rules of simple arithmetic. If the point is in the First or Fourth quadrants then $\tan^{-1}\left(\frac{y}{x}\right)$ will yield the correct value for θ . However, if the point is in either the second or third quadrants $\tan^{-1}\left(\frac{y}{x}\right)$ will **NOT** yield the correct answer, and you will need to add 180° to get the correct value for θ .

Vectors and Right Triangles

A vector is a mathematical 'object' that has both a magnitude (size), and a direction, and it requires at least two numbers to describe a vector. There are two basic forms that those two bits of information about a vector are written in; **Pure Form**, or **Component Form**.

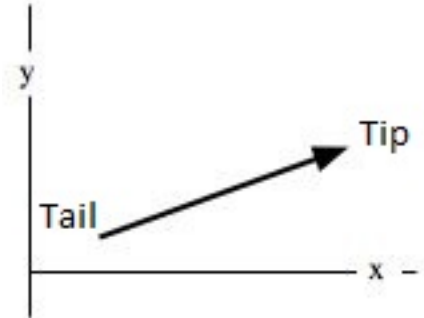
1. The **Pure Form** is called that because the two pieces of information about the vector, magnitude, and directions are completely separated from each other. So one number gives you purely information about the magnitude, and the other number gives you purely information about the direction.
 - Let's assume we have a vector \vec{V} , then in pure form it would be written as $\vec{V} = (V, \theta)$
 - Where the first number V is the magnitude, while the second number θ is the direction of the vector.
2. The **Component Form** still describes the vector by using two numbers, but the information about magnitude, and direction are sort of 'mixed up' between both of the numbers.
 - Let's assume the same vector \vec{V} as before is written in component form instead, then that would be $\vec{V} = (V_x, V_y)$.
 - The first number V_x is displacement in the x-direction, and the second number V_y is the displacement in the y-direction.
3. If you look at the same vector drawn out together in both forms a few things should pop out at you.
 - First that both forms of the vector start and stop at the same locations. That is because they are the same vector, just being described in two different ways.
 - Second, together they make a RIGHT TRIANGLE. The pure form is just the vector written in polar coordinates, and the component form is the vector written in rectangular coordinates. We can use the properties of the right triangle to convert back and forth between the two forms as needed.
 - The Pure form vector plays the role of the hypotenuse of the right triangle, and the component form plays the role of the two other legs of the right triangle.



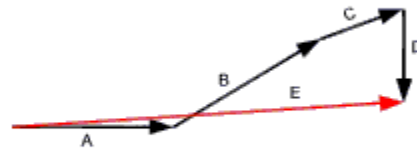
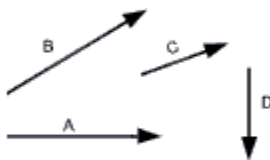
Adding Vectors

The summation of a set of vectors is called the resultant vector. We will be examining three different methods of adding vectors to obtain the resultant vector: graphical, mathematical and experimental. Regardless of the method used when adding a set of vectors, one should obtain the same resultant.

Graphically – In this method vectors are denoted by arrows, also called rays. A ray has a front end and a back end, the tip, and the tail respectively. The length of the ray represents the magnitude of the vector, the orientation of the ray, from tail to tip, is the direction of the vector. Adding vectors graphically is a rather straightforward method.



1. First you pick a scale to be used for the magnitude of the vectors you are going to add. So that 1 unit of length represents so many units of whatever magnitude the vectors have.
 - As an example 1.0 cm = 5.0 Newtons.
2. Then take one of the vectors you are adding together, and draw it with its tail at the origin of a coordinate system. The length of the vector is to be determined by the scale you chose, and its orientation is the orientation that is given for it.
3. Then you draw the next vector with its tail at the tip of the first vector. Again, its length is to be determined by the scale you chose, and its orientation is the orientation that is given for it.
4. Repeat step 3 till there are no more vectors to add.
5. Then draw a line from the origin to the tip of the last vector you added. This line is called the 'resultant', and it is the vector summation of all the vectors you added. Its length gives you the magnitude of the resultant, and its orientation, from tail to tip, gives you the direction of the resultant.



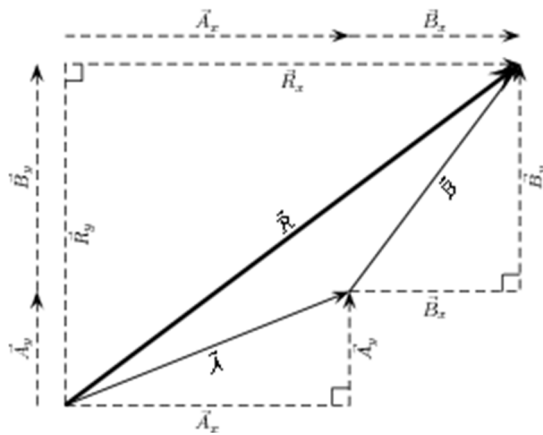
Analytical – A simple method for adding vectors mathematically is, always make sure the vectors are in their component form before you add them. The reason for this is that while

adding vectors in their pure form can get rather complicated, vector components add like simple scalars. Meaning vector components obey the rules of simple arithmetic.

Let us examine this by adding two vectors \vec{A} , and \vec{B} together to get the resultant vector \vec{R} .

$$\vec{A} + \vec{B} = \vec{R}$$

If we break down our vectors from their pure form to component form then we can simply add up all the x-components to get the x-component of the resultant, and add up all the y-components to get the y-components of the resultant.



$$A_x + B_x = R_x$$

$$A_y + B_y = R_y$$

Once you have the components of the resultant, then you simply use the properties of the right triangle to convert the resultant to the pure form.

$$\sqrt{R_x^2 + R_y^2} = R \quad \tan^{-1}\left(\frac{R_y}{R_x}\right) = \theta$$

Of course, this method can be extended to the summation of any number of vectors.

Experimentally – In this lab we will be using a device called a Force Table to find the summation of various combinations of force vectors. A Force Table is a flat, small circular table with the angular position marked all around its edges. Various amounts of masses are hung over the sides by strings, and

those strings are all connected together by a little ring loop (or plastic ring) around a little pole at the center of the Force Table. To determine what the sum of all the force vectors the masses on the strings add up to, you add additional masses to yet another string till the new force balances out the original forces. When the forces are all balanced out, the loop ring will be right at the center of the Force Table. The force that balanced out the original forces is called the Equilibrant, and it made the entire system of forces sum to zero, because the **Equilibrant has the exact same magnitude of the Resultant of the original forces, but the exact opposite direction.**

$$\vec{R} = (R, \theta) = -\vec{E} = (R, \theta \pm 180^\circ)$$

PROCEDURE - Experimental

Part I: Two Applied Forces

1. Place a pulley at the 20.0° mark on the force table and place a total of 0.050 kg on the end of the string passing over this pulley. Be sure to include the 0.005 kg of the mass holder in this total. Calculate the magnitude of the force (in Newtons) created by this mass. (Use three significant figures for all calculations in this laboratory.) Record the value of this force as F_1 in Data Table 1.
2. Place a second pulley at the 90.0° mark on the force table, using the same set of marks as the first. Place a total of 0.100 kg on the end of the string. Calculate the force produced and record as F_2 in Data Table 1.
3. Determine by trial and error where a third pulley must be located, with an appropriate mass, to set the connecting ring in equilibrium. This is achieved when the ring is centered over the pin. Assure that all strings are positioned on the ring so that each is directed toward the center of the ring. Move the ring off-center and test if it returns to the desired location when released.
4. Convert the mass determined in Step 3 to its force value, and record that value and direction (force table marker) as F_{E1} , the equilibrant.
5. From the value of the equilibrant force F_{E1} , determine the magnitude and direction of the resultant force F_{R1} , recording these two values in Data Table 1.

Part II: Three Applied Forces

1. Place a pulley at 30.0° with 0.0500 kg on its string, one at 130.0° with 0.075 kg on it, and a third at 160.0° with 0.040 kg on it. (Assure you are using the same set of degree marks.)
2. Calculate the force produced by these three masses and record them as F_3 , F_4 , and F_5 in Data Table 2.
3. Following the procedure outlined in Steps 3 through 5 in Part I above, determine the equilibrant force and resultant force, recording their magnitudes and directions in Data Table 2 as F_{E2} and F_{R2} .

Part 1: Two Applied Forces –

Procedure – Graphically, and Analytical

1. Find the resultant of these two applied forces by scaled graphical construction using the parallelogram [AKA head-to-tail] method. Using a ruler and protractor, construct vectors whose scaled length and direction represent F_1 and F_2 . For example, a convenient scale might be 1.00 cm = 0.100 N. Be careful to note that all directions are given relative to the force table, and this must be taken into account in the graphical construction to ensure the proper angle of one vector to another. Read the magnitude and direction of the resultant from your graphical solution and record them in the appropriate section of Calculations Table 1.
2. Using trigonometry, calculate the components of F_1 and F_2 and record them in the mathematical solution portion of Calculations Table 1. Add the components algebraically and determine the magnitude of the resultant by the Pythagorean Theorem. Determine the angle of the resultant from the arctangent of the components. Record these results in Calculations Table 1.
3. Calculate the percent error of the magnitude of the experimental value of F_R compared to the analytical solution of F_R . Also calculate the percent error of the magnitude of the graphical solution for F_R compared to the mathematical solution for F_R . For each of these comparisons, also calculate the magnitude of the difference in the angle. Record all values in Calculations Table 1.

Part 2: Three Applied Forces

Procedure – Graphically, and Analytical

1. Find the resultant of these three applied forces using scaled graphical construction; but, for this case, use the polygon method. Read the magnitude and direction of the resultant from your graphical solution and record them in Calculations Table 2.
2. Using trigonometry, calculate the x- and y-components of all three applied forces, add algebraically each set of components, and determine the magnitude and direction of the resultant from these.
3. Make the same error calculations for this problem as asked for in step 3 above in Part 1.

Analysis of Vector Addition Lab

Name _____ Group# _____

Course/Section _____

Instructor _____

Tables 1 (15 points)

Experimental

Force	Mass (kg)	Force (N)	Direction
F_1	0.050		20.0°
F_2	0.100		90.0°
<i>Equilibrant F_{E1}</i>			
<i>Resultant F_{R1}</i>			

Graphical Solution

Force	Force (N)	Direction
F_1		20.0°
F_2		90.0°
<i>Resultant F_{R1}</i>		

Analytical Solution

Force	Force (N)	Direction	x-component	y-component
F_1		20.0°		
F_2		90.0°		
<i>Resultant F_{R2}</i>				

Part 1: Error Calculations (15 points)

% Error of magnitude for Experimental compared to Analytical = _____ %

% Error of direction for Experimental compared to Analytical = _____ %

% Error of magnitude Graphical compared to Analytical = _____ %

% Error of direction Graphical compared to Analytical = _____ %

Tables 2 (15 points)

Experimental

Force	Mass (kg)	Force (N)	Direction
F_3	0.050		30.0°
F_4	0.075		130.0°
F_5	0.040		160.0°
Equilibrant F_{E2}			
Resultant F_{R2}			

Graphical Solution

Force	Force (N)	Direction
F_3		30.0°
F_4		130.0°
F_5		160.0°
Resultant F_{R2}		

Analytical Solution

Force	Force (N)	Direction	x-component	y-component
\underline{F}_3		30.0°		
F_4		130.0°		
F_5		160.0°		
Resultant F_{R2}				

Part 1: Error Calculations (15 points)

% Error of magnitude for Experimental compared to Analytical = _____ %

% Error of direction for Experimental compared to Analytical = _____ %

% Error of magnitude Graphical compared to Analytical = _____ %

% Error of direction Graphical compared to Analytical = _____ %

3. Two forces are applied to the ring of a force table, one at an angle of 20.0° , and the other at 80.0° . Regardless of the magnitude of the forces, will the equilibrant be in the (a) first quadrant, (b) second quadrant, (c) third quadrant, (d) fourth quadrant or (e) you cannot tell which quadrant from the available information? (8 points)
4. Two forces, one of magnitude 2.00 N and the other of magnitude 3.00 N, are applied to the ring of a force table. The directions of both forces are unknown. Which *best* describes the limitations on the magnitude of R , the resultant? (8 points)
(a) $R \leq 5N$ (b) $2N \leq R \leq 3N$ (c) $R \geq 3N$ (d) $1N \leq R \leq 5N$ (e) $R \leq 2N$
5. Suppose the same masses are used for a force table experiment as were used in Part 1, but each pulley is moved 180° so that the 0.050 kg mass acts at 200° , and the 0.100 kg mass acts at 270° . What is the magnitude of the resultant in this case? How does it compare to the resultant in Part 1? (6 points)

6. As stated earlier, the pulleys introduce a possible source of error because of their possible friction. Given that they are a source of error, why are the pulleys used at all? In other words, what is the function of the pulleys? (6 points)