EXAMPLES OF ERROR PROPAGATION FOR SPECIFIC EXPERIMENTS
Ohm's Law & Resistors

Problem: The parallel combination of three resistors R_1, R_2, and R_3 is written as:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Where $R_1 = \overline{R_1} \pm \Delta R_1$, $R_2 = \overline{R_2} \pm \Delta R_2$ and $R_3 = \overline{R_3} \pm \Delta R_3$ are the absolute uncertainties for those resistors. Determine the uncertainty in R.

Answer:

The equation for absolute uncertainties for R is:

$$\Delta\left(\frac{1}{R}\right) = \left[\left(\frac{\partial\left(\frac{1}{R}\right)}{\partial \mathrm{R}_1}\Delta \mathrm{R}_1\right)^2 + \left(\frac{\partial\left(\frac{1}{R}\right)}{\partial \mathrm{R}_2}\Delta \mathrm{R}_2\right)^2 + \left(\frac{\partial\left(\frac{1}{R}\right)}{\partial \mathrm{R}_3}\Delta \mathrm{R}_3\right)^2\right]^{\frac{1}{2}} \qquad \text{Equation AE1}$$

$$\frac{\partial\left(\frac{1}{R}\right)}{\partial R_1} = \frac{\partial}{\partial R_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = -\frac{1}{R_1^2} = -\frac{1}{\overline{R_1^2}}$$

Similarly, $\frac{\partial\left(\frac{1}{R}\right)}{\partial R_2} = -\frac{1}{\overline{R_2^2}}$ and $\frac{\partial\left(\frac{1}{R}\right)}{\partial R_3} = -\frac{1}{\overline{R_3^2}}$

Subbing these values into equation AE1, we have:

$$\Delta\left(\frac{1}{R}\right) = \left[\left(\frac{1}{\overline{R_1^2}}\Delta \mathrm{R}_1\right)^2 + \left(\frac{1}{\overline{R_2^2}}\ \Delta \mathrm{R}_2\right)^2 + \left(\frac{1}{\overline{R_3^2}}\Delta \mathrm{R}_3\right)^2\right]^{\frac{1}{2}} = \Delta \mathrm{K} \qquad \text{Equation AE2}$$

Let $\left(\frac{1}{R}\right) = k \rightarrow R = \left(\frac{1}{k}\right)$

$$\Delta \mathrm{R} = \sqrt{\left(\frac{\partial \mathrm{R}}{\partial \mathrm{k}}\Delta \mathrm{k}\right)^2}$$

$$\frac{\partial R}{\partial k} = \frac{\partial}{\partial k}\left(\frac{1}{k}\right) = -\frac{1}{k^2} \rightarrow -\frac{1}{\overline{k^2}}$$

$$\Delta \mathrm{R} = \sqrt{\left(-\frac{1}{\overline{k^2}}\Delta \mathrm{k}\right)^2} = \Delta \mathrm{k} \cdot \overline{\mathrm{R}}^2 \qquad \text{Equation AE3}$$

From Equation AE2 and AE3:

$$\Delta \mathrm{R} = \left[\left(\frac{1}{R_1^2}\Delta \mathrm{R}_1\right)^2 + \left(\frac{1}{R_2^2}\ \Delta \mathrm{R}_2\right)^2 + \left(\frac{1}{R_3^2}\Delta \mathrm{R}_3\right)^2\right]^{\frac{1}{2}} \cdot \overline{\mathrm{R}}^2$$

$$\frac{\Delta \mathrm{R}}{\bar{R}} = \left[\left(\frac{\bar{R}}{R_1^2}\Delta \mathrm{R}_1\right)^2 + \left(\frac{\bar{R}}{R_2^2}\ \Delta \mathrm{R}_2\right)^2 + \left(\frac{\bar{R}}{R_3^2}\Delta \mathrm{R}_3\right)^2\right]^{\frac{1}{2}}$$