

## RC and LRC Circuits

### Equipment

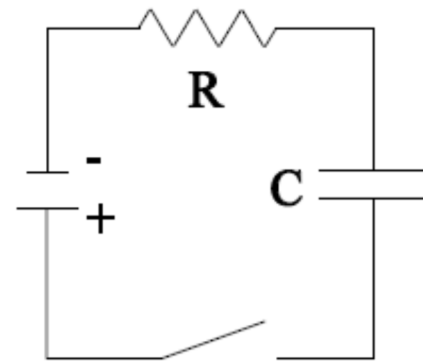
Qty	Items	Part Number
1	AC/DC Electronics Laboratory	CI-6503
1	Voltage Sensor	
2	Banana Plugs, Patch Cord	
1	330 $\mu\text{F}$ Capacitor	
1	1000 $\Omega$ resistor	
1	Iron core	
1	100 $\mu\text{F}$ Capacitor	
1	10 $\Omega$ resistor	

### Purpose: RC Circuits

The purpose of the first part of this activity is to examine some of the basic properties, and behavior of a simple circuit that contains a resistor, and a capacitor that are connected to each other in series. The other two standard components of an RC circuit are a voltage source, and a switch to open and close the circuit.

### Theory: RC Circuits

When a DC voltage source is connected across an uncharged capacitor, a current begins to flow in the circuit, and the capacitor begins to gain electrical charge. The rate at which the capacitor gains electrical charge decreases as time goes on till eventually the capacitor is said to be fully charged, and there is no longer any current flowing in the circuit. This occurs because as electrical charges accumulate on the plates of the capacitor the voltage source must do more and more work to cause additional charge to build up on the plates. Which results in the charges building up at slower, and slower rates till the voltage source can't do enough work to get anymore charge to build up, the current is said to stop flowing, and the capacitor is said to be full. **The amount of electrical charge in a capacitor, as a function of time, that is being charged by a voltage source is given by the equation;**



$$q(t) = q_{max} \left( 1 - e^{-t/\tau} \right)$$

In this equation it is assumed that time is set to zero,  $t = 0$ , when you begin to charge the capacitor, and that the capacitor is initially 'empty' of charge  $q(0) = 0$ . Also,  $q_{max}$  is the maximum charge the plates can hold, and  $\tau$  (tau) is the **Time Constant of the RC circuit itself  $\tau = RC$  where  $R$  is the resistance, and  $C$  is the capacitance.** The Time Constant has the SI units of seconds. The Time Constant is considered to be the fundamental characteristic of an RC circuit due to an interesting physical result that occurs when the amount of time spent charging a capacitor is equal to the Time Constant  $t = \tau$ . Inserting this into our equation for charging a capacitor gives the following:

$$q(\tau) = q_{max}(1 - e^{-\tau/\tau})$$

$$q(\tau) = q_{max}(1 - e^{-1})$$

$$q(\tau) = q_{max} \cdot 0.63$$

As you can see, regardless of the values of either the resistance or capacitance of the circuit, the capacitor will be 63% charged when the amount of time spent charging it is equal to the Time Constant of the circuit.

When a DC voltage is removed from a fully charged capacitor, while the circuit remains closed, the electric charge that is contained on the capacitor starts to redistribute itself, which results in a current beginning to flow through the circuit. As time goes on, the rate at which the charge redistributes itself gets slower and slower till finally the electric charge is said to be full discharged from the capacitor, and the current is said to stop flowing. **Assuming we start with a fully charged capacitor  $q(t) = q(0) = q_{max}$  then the amount of electric charge still in a discharging capacitor, as a function of time, is given by:**

$$q(t) = q_{max} \cdot e^{-t/\tau}$$

Just like when charging a capacitor, when discharging a RC circuit, something interesting occurs when the amount of time spent discharging it is equal to the Time Constant of the circuit. Inserting  $t = \tau$  yields:

$$q(\tau) = q_{max} \cdot e^{-\tau/\tau}$$

$$q(\tau) = q_{max}e^{-1}$$

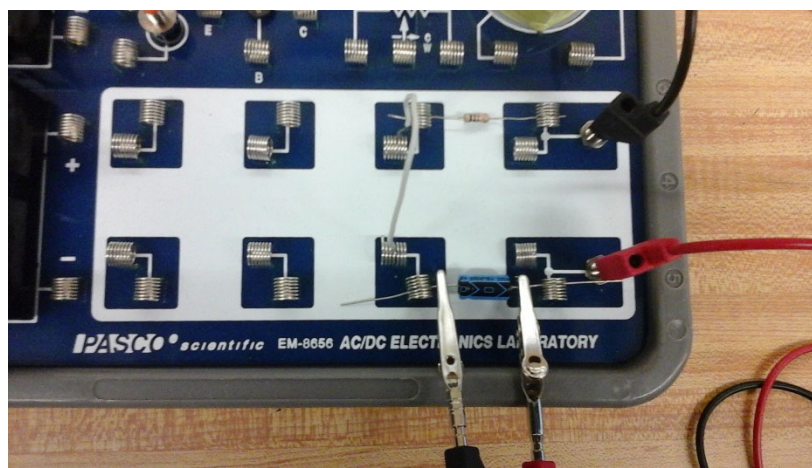
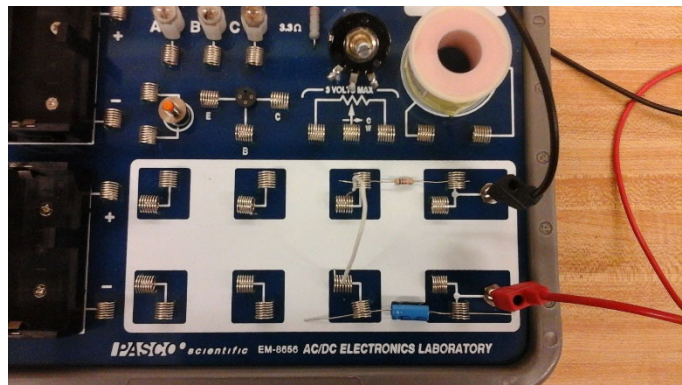
$$q(\tau) = q_{max} \cdot 0.37$$

As you can see, regardless of the values of either the resistance or the capacitance of the circuit, the capacitor will only have 37% of the maximum charge left in it when the amount of time spent discharging it is equal to the Time Constant of the circuit.

## Setup: RC Circuit

1. Open the Capstone software. On the left side of the main screen is the Tool Bar. Click on the Hardware Setup icon. This will open the Setup window.
  - Click on Channel A of the picture of the 850 Universal Interface in the Setup window, and then scroll down, and add the 'Voltage Sensor'.
2. On the bottom center left of the main screen, the Sample Rate Tab should now say 'Voltage Sensor'.

- Set the sample rate to 50 Hz.
3. To the right of the Sample Rate Tab is the Recording Conditions Icon. Click on it, and the Recording Conditions window should open.
    - Click on 'Stop Conditions'.
    - Change 'Conditions Type' to 'Time Based'.
    - Set 'Record Time' to 20 seconds.
    - Click 'ok', and the window should close.
  4. In the Tool Bar, now click on the Signal Generator Icon. This will open the Signal Generator window.
    - In the Signal Generator window click on the tab "850 output 1" tab. This will open up the options window for the output generator 1.
    - Set the Waveform to 'Positive Square Wave'.
    - Set the frequency to 0.100 Hz.
    - Set the Amplitude to 1 Volt.
    - Set the Generator to 'Auto', so that it will start and stop automatically when you start and stop collecting data.
  5. Close the Tool Bar.
  6. On the right side of the main screen is the Display Bar.
    - Left click, and hold on the Graph Icon. Then drag out to the middle of the main screen, and release.
    - Click on the y-axis 'select measurement' tab, and select Voltage. The computer should automatically select time for the x-axis.
  7. Plug in the Voltage Sensor to Analog Channel A of the 850 Universal Interface.
  8. Plug in two Patch Cords to the Output Generator 1 of the 850 Universal Interface (top right of the interface). Then plug the other ends of the Patch Cords to the AC/DC Electronics Laboratory.
  9. Using a 1000  $\Omega$  resistor, a 330  $\mu\text{F}$  capacitor, and one jumper cable, construct the simple RC circuit shown in the picture. Make sure the capacitor is pointing towards the negative terminal
  10. Attach a Banana Clip to each of the two ends of the Voltage Sensor.
  11. Then Connect the Voltage Sensor to the capacitor with one end of the Voltage Sensor on either



side of the capacitor as shown in the picture.

### Procedure: RC Circuit

1. At the bottom of the main screen click on "Record", then wait for the 20 seconds it takes for the experiment to run its course.

### Analysis: RC Circuit

1. Expand a region of the graph display so that you can easily see a region of the plot of Voltage vs. Time graph that shows the voltage rising from zero volts to the maximum voltage.
2. Use the coordinate tool to find the time value of the time where the voltage starts to increase from zero. Record this time in Table 1, as  $t_o$  for charging.
3. Then use the coordinate tool to find the time that the voltage is 63% of the maximum voltage. Record this time in Table 1, as  $\tau'_{exp}$  for charging.
4. Expand a region of the graph display so that you can easily see a region of the plot of Voltage vs. Time graph that shows the voltage falling from maximum volts to zero volts.
5. Use the coordinate tool to find the time value of the time where the voltage starts to decrease from maximum voltage. Record this time in Table 1, as  $t_o$  for discharging
6. Then use the coordinate tool to find the time that the voltage is 37% of the maximum voltage. Record this time in Table 1, as  $\tau'_{exp}$  for charging.
7. Save the graph, so it can be turned in with your lab report.

### Purpose: LRC Circuits

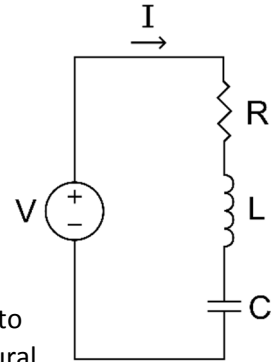
The Purpose of this activity is to study the resonance in an inductor-resistor-capacitor circuit (LRC circuit) by examining the current through the circuit as a function of the frequency of the applied voltage. Determine what happens to the amplitude of the current wave in the LRC circuit when the frequency of the applied voltage is at or near the resonance frequency of the circuit. As always, the voltage measured across the resistor is related to the current through the resistor by:

$$I = \frac{V_R}{R}$$

Where  $V_R$  is the instantaneous voltage across the resistor, and  $R$  is the resistance of the resistor. We will also examine the phase relationship between the applied voltage and the resistor voltage as the frequency varies.

### Theory: LRC Circuit

When a vibrating mechanical system is set in motion, it vibrates at its natural frequency. However, a mechanical system can be forced to vibrate at a different frequency. The amplitude of vibration, and hence the energy transferred to the system, depends on the difference between the natural frequency and the frequency of the forced vibration. The amplitude becomes very large when the difference between the natural and forced frequency of the system is sometimes called the resonant frequency. At resonance, relatively little energy is required to get a large amplitude. One example of resonance is when a singer's voice is used to shatter a glass. When the frequency of the note the singer sings matches the natural frequency of the glass, the glass will start to vibrate. Since the glass is vibrating at its natural frequency its amplitude of oscillation will be relatively large. If it becomes large enough the glass will shatter.



Electronic resonance is analogous to mechanical resonance. The energy is most efficiently transferred to a system at the system's resonance frequency, thereby maximizing the amplitude of oscillation of the system. The amplitude of the AC current  $I_o$  in a simple LRC circuit is dependent on the amplitude of the applied voltage  $V_o$  and the impedance  $Z$ .

$$I_o = \frac{V_o}{Z}$$

Since the impedance depends on the frequency the current varies with frequency by:

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

Where  $X_L$  = inductive reactance =  $\omega L$ ,  $X_C$  = capacitive reactance =  $\frac{1}{\omega C}$ ,  $R$  = resistance, and  $\omega$  = angular frequency =  $2\pi f$ . The current will be maximum when the circuit is driven at its resonant frequency:

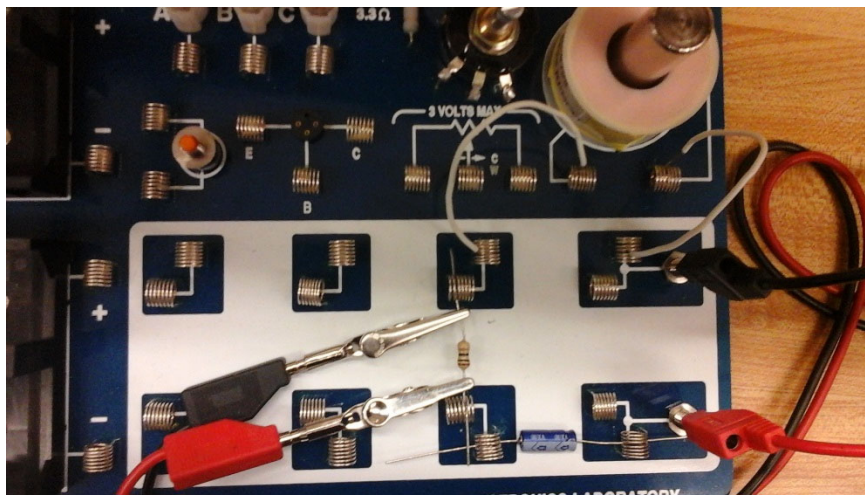
$$\omega_r = \frac{1}{\sqrt{LC}}$$

For at resonant frequency  $X_L = X_C$ , and therefore  $Z = R$ .

**Setup: LRC Circuit** (The LRC setup instructions are written with the assumption everything is being set up anew.)

1. Open the Capstone software. On the left side of the main screen is the Tool Bar. Click on the Hardware Setup icon. This will open the Setup window.

- Click on Analog Channel A of the picture of the 850 Universal Interface in the Setup window, and then scroll down, and add the 'Voltage Sensor'.
  - Click on Output Channel 1 of the picture of the 850 Universal Interface in the Setup window, and then scroll down, and add the 'Output Voltage-Current Sensor'.
2. On the bottom center left of the main screen the Sample Rate Tab should now say 'Voltage Sensor'.
    - Set the Sample Rate to 5000 Hz (5 kHz)
  3. In the Tool Bar, now click on the Signal Generator Icon. This will open the Signal Generator window.
    - In the Signal Generator window click on the tab "850 output 1" tab. This will open up the options window for the output generator 1.
    - Set the Waveform to a "Sine" wave.
    - Set the frequency to 10 Hz.
    - Set the Amplitude to 2.97 volts.
    - Set the Generator to "Auto", so that it will start and stop automatically when you start and stop collecting data.
  4. At the top right of the Tool Bar, click on the push pin icon to rescale the main window so you can keep the tool bar open while viewing the whole graph.
  5. On the right side of the main screen is the Display Bar.
    - Left click, and hold on the Scope Icon. Then drag out to the middle of the main screen, and release.
    - Click on the y-axis 'select measurement' tab, and select 'Voltage, Ch A(V)'. The computer should automatically select time for the x-axis.
    - Then on the top tool bar of the Scope Screen click on the "Add new y-axis to scope display" tab.
    - Click on the new y-axis 'select measurement' tab, and select 'Output Voltage, Ch 1(V)'.
  6. Plug in the Voltage Sensor to Analog Channel A of the 850 Universal Interface.
  7. Plug in two Patch Cords to the Output Generator 1 of the 850 Universal Interface (top right of the interface). Then plug the other ends of the Patch Cords to the AC/CD Electronics Laboratory.
  8. Using a 100  $\mu\text{F}$  capacitor, a 10  $\Omega$  resistor, and 2 jumper cords, construct the simple LRC circuit as shown in the picture. Make sure the capacitor is pointing towards the negative terminal.
    - Record their values in Table 2.2
  9. Attach a Banana Clip to each of the two ends of the Voltage Sensor'.
  10. Put the Iron Core inside the inductor coil.
  11. Then Connect the Voltage Sensor to the resistor with one end of the Voltage Sensor on either



side of the resistor as shown in the picture.

### Procedure: LRC Circuit

1. At the bottom of the main screen click on "Record".
2. In the Scope display, using the coordinate tool, determine the maximum value of the Voltage across the resistor  $V_R$  (Voltage Ch A).
  - Record  $V_R$  in the Table 2.1 for 10 Hz.
3. In the Tool Bar click on the signal generator icon.
  - Change the frequency to 20 Hz, then repeat steps 1 and 2 for 20 Hz.
4. Keep increasing the frequency rate by increments of 10 Hz, and repeating the process till the last run is completes is for 150 Hz.
  - Adjust the Sweep Speed in the Scope display as needed.
5. Look at the date in Table 2.1, and estimate the approximate resonant frequency. It will be where the voltage across the resistor reaches its maximum value.
6. Adjust the function generator to the value of your estimate of the resonant frequency. Make small adjustments to the frequency until the trace of voltage from the resistor is *in phase* with the trace of the Output voltage.
  - Record your experimental resonant frequency in Table 2.1 as ' $f_r$ '.

## Analysis of RC and LRC Circuits Lab

Name \_\_\_\_\_ Group# \_\_\_\_\_

Course/Section \_\_\_\_\_

Instructor \_\_\_\_\_

### RC Circuits:

**Table 1 (5 points)**

	Charging	Discharging
$t_o$		
$\tau'_{exp}$		
$\tau_{exp}$		

1. Calculate your experimental value of the Time Constant,  $\tau_{exp}$ , obtained by both charging and discharging the capacitor by taking the difference of  $\tau'_{exp} - t_o$  and record it as  $\tau_{exp}$  in Table 1, under charging and then discharging. Show work. (4 points)
  
2. Calculate the theoretical value for  $\tau$ . Show work. (2 points)
  
3. Calculate the % errors for the Time Constant for both the charging and discharging values. Show work. (4 points)
  
4. Calculate the % difference between your experimental values of the Time Constant. Show work. (5 points)



## LRC Circuits:

**Table 2.1 (15 points)**

<i>Freq (Hz)</i>	<i>V<sub>R</sub>(V)</i>	<i>V<sub>R</sub>/R (A)</i>	<i>Freq(Hz)</i>	<i>V<sub>R</sub>(V)</i>	<i>V<sub>R</sub>/R (A)</i>
10			90		
20			100		
30			110		
40			120		
50			130		
60			140		
70			150		
80			$f_r$		

Complete Table 2.1 by calculating the current passing through the resistor for each frequency, and record the values. Show work.

**Table 2.2 (2 points)**

Item	Value
Inductance	30 mH
Resistance	
Capacitance	

**Table 2.3 (5 points)**

Experimental Resonant angular frequency, $\omega_r$	
Theoretical resonant angular frequency	

1. In Excel, or some other graphing program, from the data from Table 2.1, plot Current vs. Frequency. (15 points)
2. The Current vs. Frequency graph should be nearly symmetrical about a particular frequency. What frequency is it nearly symmetrical about, and why? (5 points)

3. Using the experimental resonant frequency,  $f_r$ , calculate the resonant angular frequency,  $\omega_R = 2\pi f_r$ . Then using the values in Table 2.2 for inductance and capacitance calculate the theoretical resonant frequency. Show work, and put your answers in Table 2.3 (5 points)
  
4. What is the % error between your measured resonant angular frequency, and the theoretical resonant angular frequency? (3 points)
  
5. At resonance, the reactance of the inductor and the capacitor cancel each other out so that the impedance ( $Z$ ) is equal to just the resistance ( $R$ ). Calculate the impedance ( $Z$ ) and compare this value to the resistance of the circuit by using the amplitude of the current at resonance in the equation  $R = V/I$  (where  $V$  is the amplitude of the applied voltage). Is this resistance equal to  $10 \Omega$ ? Why not? (10 points)
  
6. Name at least two real world applications of inductors. (10 points)
  
7. Did our experiments confirm the theories? Justify your answer. (10 points)