Rotational Inertial

Equipment

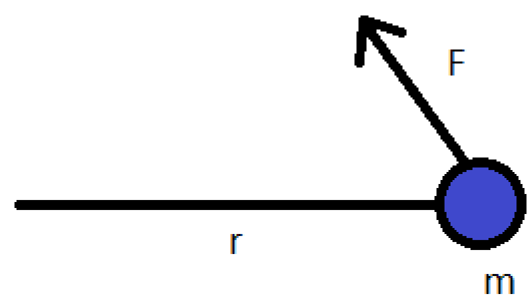
Qty	Item	Parts Number
1	Rotational Motion Sensor	CI-6538
1	Rotational Inertia Set: ring and disk	ME-8953
1	Large Rod	ME-8977
1	Universal Table Clamp	ME-9376B
1	Detectable Pulley	ME-9448B
1	Vernier Caliper	
1	Mass and Hanger Set	ME-8979
1	Padding	

Purpose

The purpose of this exercise is to examine the moment of inertia of both a ring and disk, and to experimentally confirm that the moment of inertia of an object is a function of both its mass and how that mass is spatially distributed.

Theory

Let us assume there is a mass m, initially at rest, that is attached to one end of a massless rod of length r, and the other end of the rod is attached to a frictionless pivot that is free to rotate 360°. If one were to apply a net force F to the mass it would induce rotational motion about the pivot point, but only the component of the applied for force that is tangential (at a right angle) to the length r would contribute to the change in motion of the mass. By Newton's Second Law we know the tangential force would be related to the tangential acceleration by the following equation:



$$F_T = ma_T$$

The torque acting about the point of rotation that is associated with the tangential component of the applied force would be given by:

$$\tau = F_T r = mra_T$$

We also know that the angular acceleration that the mass is experiencing is related to its tangential acceleration by $a_T = r\alpha$, so we can insert that into the equation, giving us:

$$\tau = mr^2\alpha$$

What would happen if you would apply a force to a rigid body (solid object) that is fixed in location, but free to rotate about an axis? Now a rigid body is just a collection of the point masses that make the total mass M of the rigid body. Each of those point masses m_i will have its own direct line distance r_i from the axis of rotation and itself. So for each point mass we could go through the exact same argument as we just went through to arrive at a similar equation;

$$\tau_i = m_i r_i^2 \alpha$$

The infinitesimal amount of toque τ_i for each point mass is the product of the term $m_i r_i^2$ and α the angular acceleration of the rigid body. The angular acceleration does not have a subscript on it because as a rigid body all the point masses rotate together, so all the point masses have the exact same angular acceleration. To find the total toque acting on the entire rigid body you simply sum up all the torques acting on all the point masses.

$$\tau = \sum_i^n \left(m_i r_i^2\right) \alpha$$

The summation term is given a name. It is called the moment of inertia of the rigid body, and its SI units are kg·m^2.

$$I = \sum_i^n \left(m_i r_i^2\right)$$

This allows us to rewrite our equation as

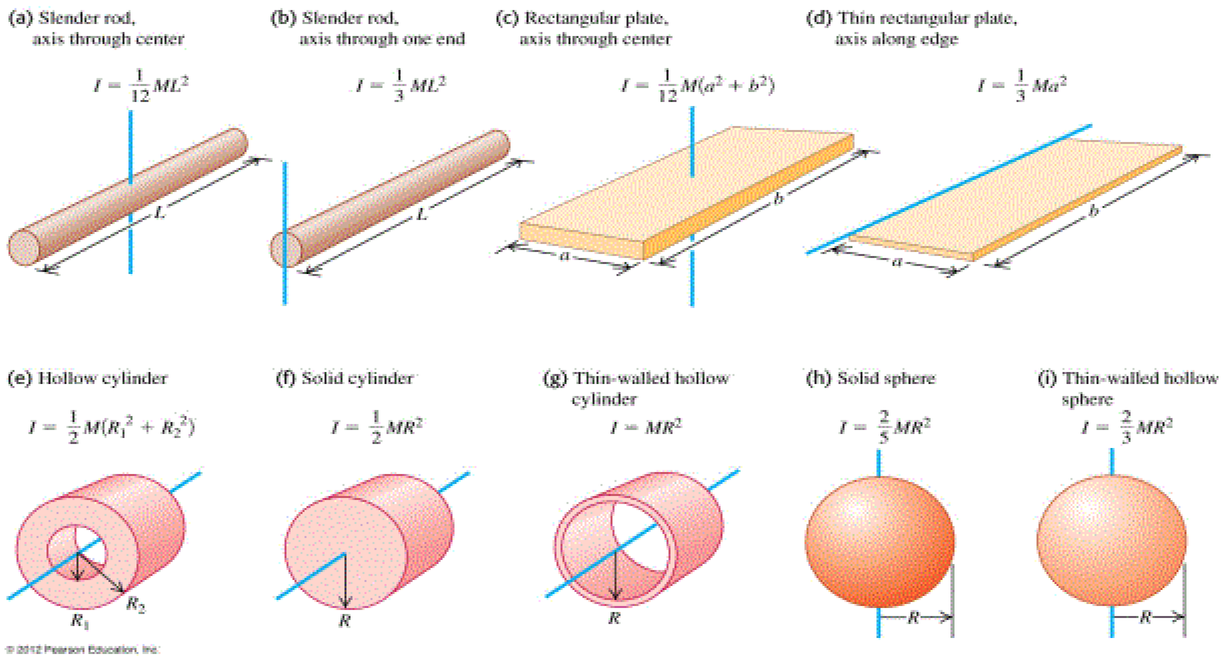
$$\tau = I\alpha$$

The moment of inertia is a quantification of how difficult (or easy) it is to get an object to change its current state of rotational motion about a particular axis or rotation. The value of the summation of the term $m_i r_i^2$ will depend on the total mass of the rigid body, its shape, and the axis of rotation that is picked. However, the summation will always have the following basic algebraic form:

$$I = \sum_i^n \left(m_i r_i^2\right) = CMr^2$$

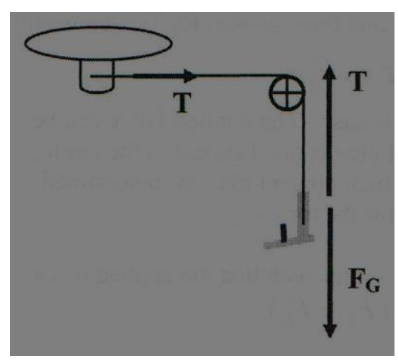
Here M is the total mass of the object, r is the 'radius' of the object, and C is a coefficient dependent on the shape of the object. A chart of the moment of inertia of some basic geometric shapes with uniform mass is given.

Table 9.2 Moments of Inertia of Various Bodies



Note: a hollow cylinder is also known as a ring. The difference between the hollow cylinder and the thin-walled cylinder is that for the thin-walled cylinder the outer and inner radii are so close in value that they can be treated as being of the same value, while for the regular hollow cylinder that isn't true.

One way to measure the moment of inertia of a rigid body experimentally is to attach it to a fixed pivot point allowing for rotation in the horizontal plane, apply a known constant torque to it, then measure its tangential acceleration, and finally do a little algebra. One step up that can be used to do this is a two pulley system. One horizontal pulley centered about the pivot point, and another vertical pulley. A string will be wrapped around the first pulley, and then strung over the vertical pulley with a known mass m_h attached to it. The mass will be released, causing a constant tension in the string, which will in turn cause a constant torque about the first pulley, finally resulting in the system experiencing a constant acceleration.



In such a setup we start with the basic equation relating the applied torque to the moment of inertia.

$$\tau = I\alpha$$

So the moment of inertia will be the torque divided by the angular acceleration of the system.

$$I = \frac{\tau}{\alpha}$$

From the free body diagram, and force summation equations of the system we see that the tension in the string will be:

$$T = m_h g - m_h a_T = m_h(g - a_T)$$

Inserting this for the tension, as well as the known equations for torque and angular acceleration, gives us:

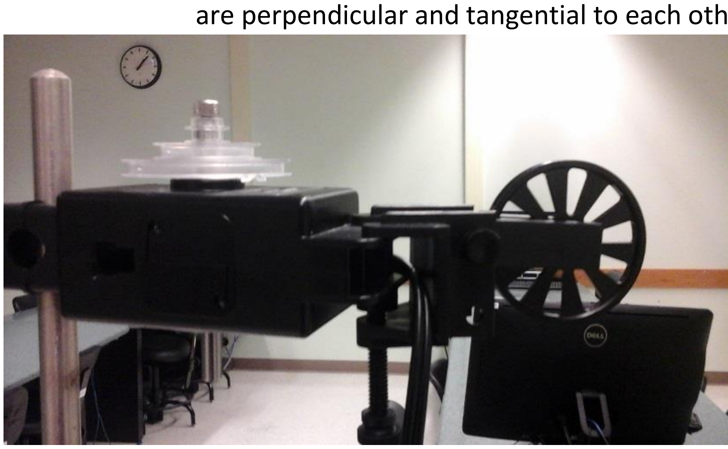
$$I = \frac{r \cdot m_h(g - a_T)}{\frac{a_T}{r}} = r^2 \cdot m_h \left(\frac{g}{a_T} - \frac{a_T}{a_T}\right)$$

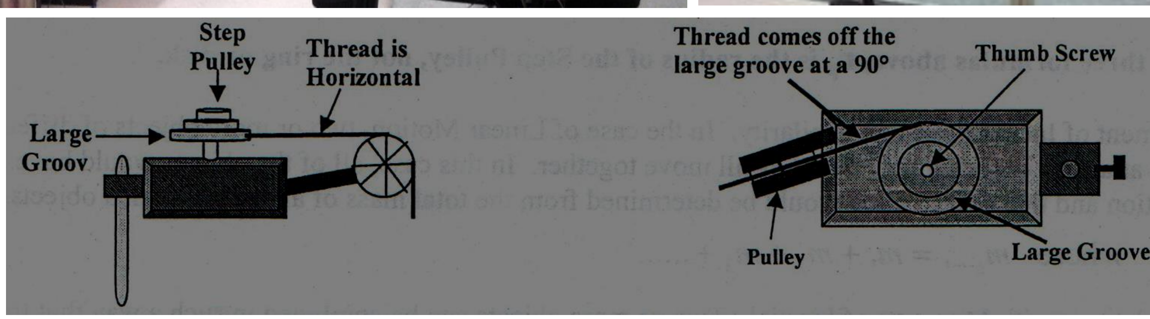
$$I = r^2 \cdot m_h \left(\frac{g}{a_T} - 1\right)$$

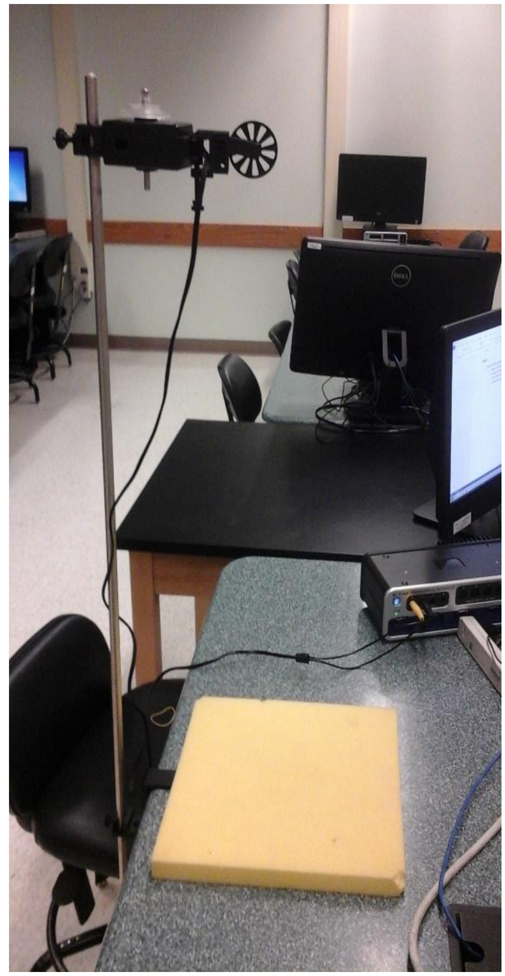
This final equation gives the experimental value for the moment of inertia of the rigid body. Where here r is the radius of the horizontal pulley the rigid body is attached to. (Please note that in constructing this equation we assumed that the moment of inertia of the two pulleys are so small compared to the moment inertia of the rigid body's that they were simply ignored.)

Setup

1. Using the listed equipment, construct the setup as shown.
   - Make sure that the step pulley is positioned on top.
   - To ensure that the torque equation reduces from $\tau = r\,F\sin\theta$ to $\tau = rF$ you need to ensure that the detectable pulley is aligned to the step pulley such that the two pulleys are perpendicular and tangential to each other, as shown in the pictures provided.







- The large pulley (the bottom pulley) should already have a string attached to it. If it does not, get a piece of string about 1.5 m in length and attach it to the large pulley.
- Attach the other end of the string to a hook from the mass and hook set.

2. Make sure the PASCO 850 Universal Interface is turned on and connected to the computer.
3. Double click the Capstone icon to open the Capstone software.
4. In the Tool Bar, on the right side of the screen, click on Hardware Setup to open the Hardware Setup window.
5. In the Hardware Setup there should be an image of the PASCO 850 Universal Interface. If there is, skip to set 6.
    - If there isn't click on Choose Interface to open the Choose Interface window. Select PASPORT, then Automatically Detect, and then click OK.
6. On the image of the PASCO 850 Universal Interface click on Digital Inputs Ch (1) to open the digital senor list.
    - Scroll down the list and select Rotary Motion Sensor. You should now see the rotary motion sensor icon connected to digital inputs Ch (1), and Ch (2).
    - Plug in the rotary motion sensor. Yellow Ch (1), and black Ch (2).
7. In the Tool Bar click on Data Summary to open up the Data Summary window.
8. In the Data Summary window click on the properties icon right to the right of where it reads Rotary Motion Sensor to open the properties window for the rotary motion sensor.
    - In the properties window for Linear Accessory, select Large Pulley (Groove), then click Ok.
9. At the bottom of the main screen make sure the sample rate for the rotary motion sensor is set to 20 Hz.
10. Close the Tool Bar.
11. In the Display Bar, on the right side of the screen, double click the Graph icon to open up a graph.
    - Click Select Measurement for the y-axis of the graph, and select Velocity (m/s).
    - The computer will then automatically select time (s) for the x-axis.

Procedure

1. Using the Vernier caliper, measure the diameters of the large grove pulley, the disk, and both the inner and outer diameters of the ring. Record the diameters in the table provided.
    - When measuring the diameter of the large grove pulley make sure to NOT include the lips around the pulley, but to insert the Vernier caliper between the lips to measure the diameter of just the pulley itself.
2. Using a mass scale, measure the masses of the ring and disk, then record their masses in the table provided.
3. Remove the thumb screw from the top of the step pulley, put the disk in place, and then reattach the thumb screw to secure the disk in place.
4. Place 25 g on the hook, and record the total mass (30 g) of the mass and hook
5. Now rotate the Step Pulley such that the string wounds up around the large groove pulley, but with enough string left over so that the hook and mass can still be strung right over the detachable pulley.
6. Now near the bottom left of the screen click on Record to start recording data, and then let go of the hanging mass.
    - Once the mass hits to padding, click on Stop to stop recording data.
7. Near the top left of the graph click on the Highlight Range icon to make a highlight box appear on the graph.
    - Move and rescale the highlight box so only the portion of the graph that represents when the hanging mass was falling is selected.
8. Near the top left of the graph click on the down arrow next to the Apply Select Curve icon to make the select curve list appear.
    - Select linear fit, and the equation for the linear fit of the highlighted portion of the graph should appear on the screen.
    - Record the slope as the acceleration of the disk in the provided table. If the slope is negative just record the magnitude of the slope. (The slope will be positive or negative depending on which direction you wound the string around the pulley.)
9. Attach the ring to the disk, and then repeat steps 5 through 8 for the combination of the two masses, and record that as the acceleration of the disk and ring in the provided table.

<u>Analysis of Rotational Inertia Lab</u>

Name____ Group#________

Course/Section____

Instructor____

Tables (20 points)

	Diameter (cm)	Radius (cm)	Radius (m)
Large grove pulley			
Disk			
Ring, inner			
Ring, outer			

	m (g)	m (kg)
Disk		
Ring		
Hanging mass		

	a (m/s^2)
Disk	
Disk and Ring	

1. Calculate the theoretical moment of inertia of the disk, and show work. (5 points)

2. Calculate the theoretical moment of inertia of the ring, and show work. (5 points)

3. Calculate the experimental moment of inertia of the disk, and show work. (5 points)

4. Calculate the experimental moment of inertia of the ring, and show work. (5 points)

5. Using the theoretical value as the accepted value calculate the % error for the disk, and show work. (10 points)

6. Using the theoretical value as the accepted value calculate the % error of the right, and show work. (10 points)

7. If you repeated this experiment with a larger hanging mass, should that change the values you would obtain for the moment of inertia of the ring and disk? Justify your answer. (10 points)

8. Why are you able to calculate the experimental value of the moment of inertia of the ring in the manner you did? (10 points)

9. Describe, in terms of energies, what is happening during the experiment. (6 points)

10. What are some reasons that account for our percent error? (6 points)

11. Do the results of our experiment confirm theoretical predictions? On what do you base your answer? (8 points)